Astrophysical background of gravitational waves: from cosmology towards the era of precision astrophysics

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in collaboration with C. Pitrou, J.P. Uzan, I. Dvorkin, R. Durrer, P. Ferreira, D. Alonso

based on

GC et al. Phys.Rev. D96 (2017) 103019
GC et al. PRL 120 (2018) 231101
GC et al. Phys.Rev. D97 (2018) 123527
GC et al. Phys.Rev. D99 (2019) 023534
GC, Dvorkin et al. Phys.Rev. D100 (2019) 063004
GC, Dvorkin et al. MNRAS Lett (2019)
Pitrou, **GC**, Uzan, arxiv 2019
Alonso, **GC**, Pitrou, Ferreira arxiv 2020
GC, Alonso, Ferreira, 2020

Outline of this talk

- **Introduction**. Astrophysical gravitational wave background: what is it? State of the art of observation & theory

- **Theoretical framework** to study anisotropies and polarization

- **Numerical predictions** in LISA and LIGO-Virgo bands

- **Astrophysical interest**: content of this new observable and what we can learn out of it

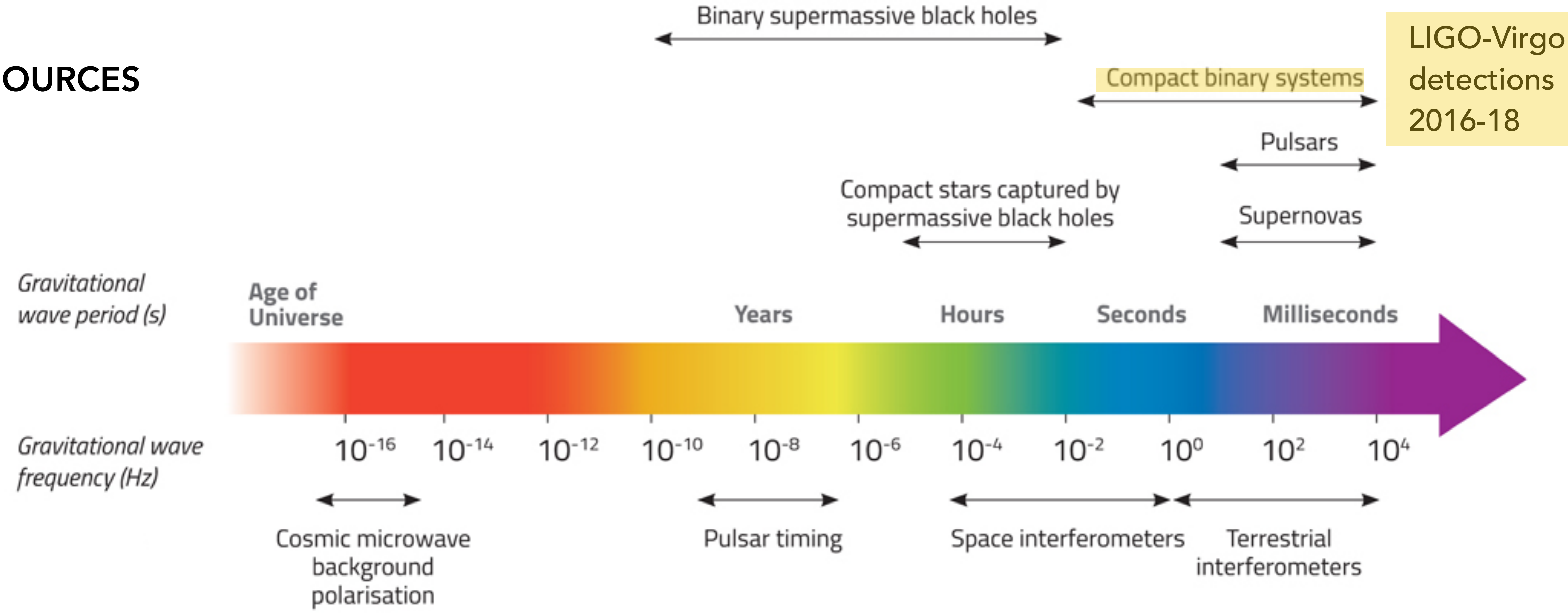
- **Ongoing work**: characterization of different noise components

(most of) **my research over last 3yr:** from theory to forecasts of detectability

The new era of gravitational wave astronomy

SOURCES

LIGO-Virgo first detections 2016-18



Binary supermassive black holes

Compact binary systems

Pulsars

Compact stars captured by supermassive black holes

Supernovas

Gravitational wave period (s)

Age of Universe

Years

Hours

Seconds

Milliseconds

Gravitational wave frequency (Hz)

10^{-16} 10^{-14} 10^{-12} 10^{-10} 10^{-8} 10^{-6} 10^{-4} 10^{-2} 10^{0} 10^{2} 10^{4}

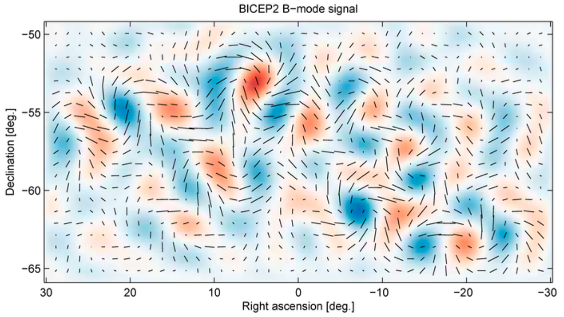
Cosmic microwave background polarisation

Pulsar timing

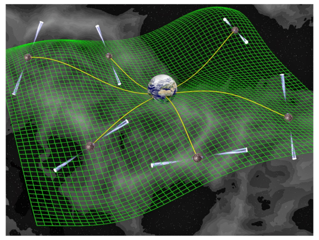
Space interferometers

Terrestrial interferometers

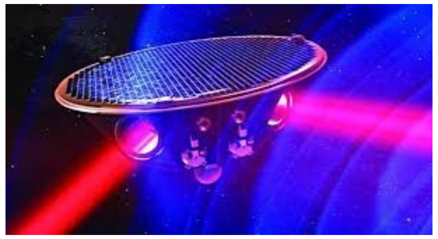
DETECTORS



CMB polarization



PTA

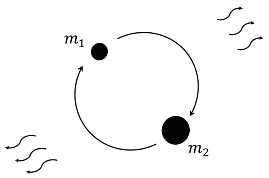
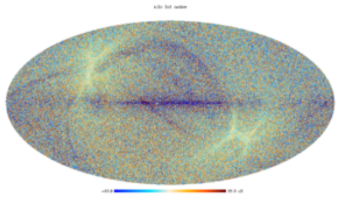


LISA (>2030)

LIGO, Virgo,LIGO-India, Kagra, ET (>2030)

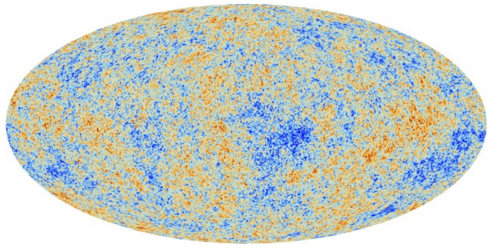
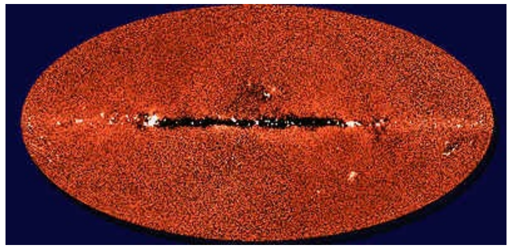
Expected detection of new astrophysical sources, and **detection of GW background**

Two types of astrophysical GW observables

astrophysical observable	what we measure	what we can learn
GW from resolvable sources 	**waveform** as a function of time and frequency	properties of the source (e.g. nature, mass, spin, distance…)
Astrophysical background of GW 	**intensity and polarization** as functions of direction and frequency	**collective properties** of a population of sources (redshift distribution, time evolution, distribution of masses…)

Stochastic backgrounds of radiation

Stochastic background: incoherent superposition of signals from all sources

	cosmological origin	astrophysical origin
EM radiation	CMB  Plank CMB map	IR extragalactic background (CIB)  Plank IR map
GW radiation	cosmological background	**astrophysical background** foreground for cosmological GW background

Astrophysical stochastic background

$$\Omega_{GW}(f) = \frac{f}{\rho_c}\frac{d\rho_{GW}(f)}{df} = \frac{f}{\rho_c}\int dz\,\frac{dE(z)}{df}\,R(z)\,\frac{n(z)}{H(z)}$$

density of sources

spectrum

rate of events

$\rho_c = cnst$

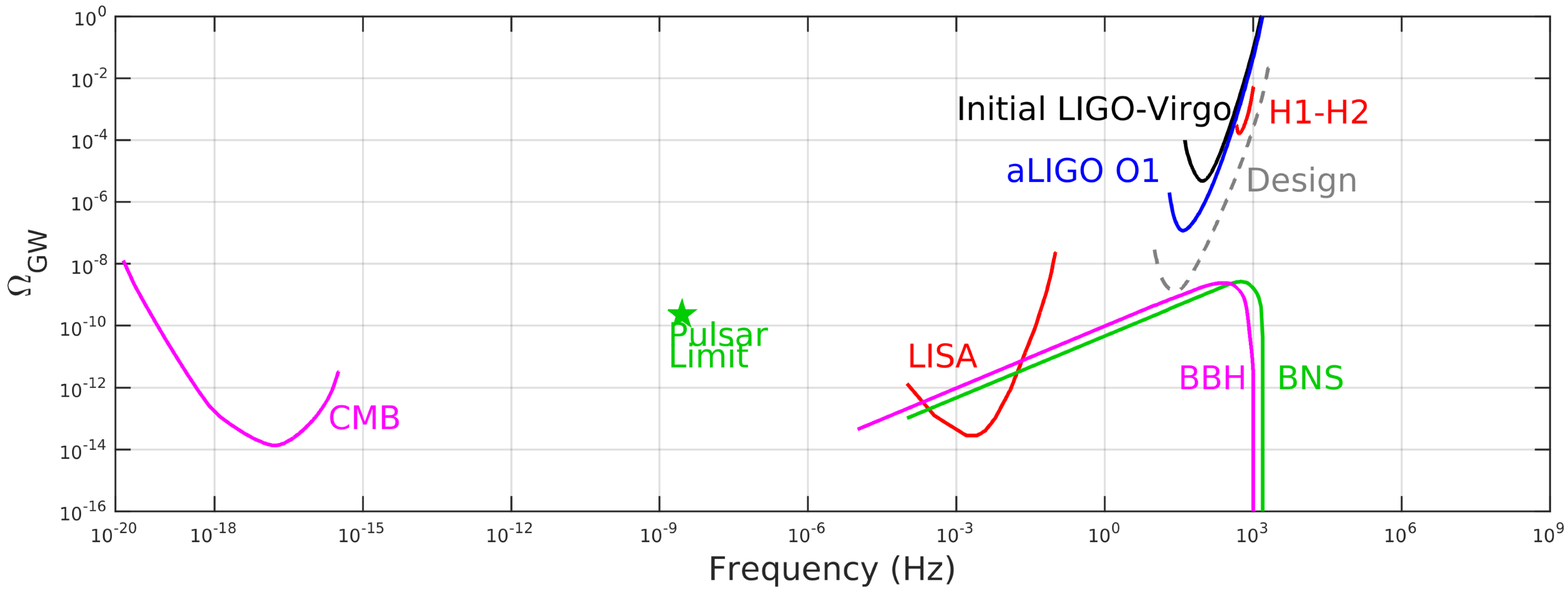
ρ_{GW} energy density background

Astrophysical stochastic background

$$\Omega_{GW}(f) = \frac{f}{\rho_c}\frac{d\rho_{GW}(f)}{df} = \frac{f}{\rho_c}\int dz\,\frac{dE(z)}{df}\,R(z)\,\frac{n(z)}{H(z)}$$

density of sources

spectrum rate of events



Abbott et al, PRL 118 (2017) 121101

upper bounds

$$\boxed{\begin{array}{c} < 1.3 \cdot 10^{-9} \\ 2.8 \times 10^{-9}\mathrm{Hz} \end{array}}$$

Shannon et al., 2013

$$\boxed{\begin{array}{l} < 4.8 \cdot 10^{-8} \\ 25\mathrm{Hz} \end{array}}$$

LIGO-Virgo
2019

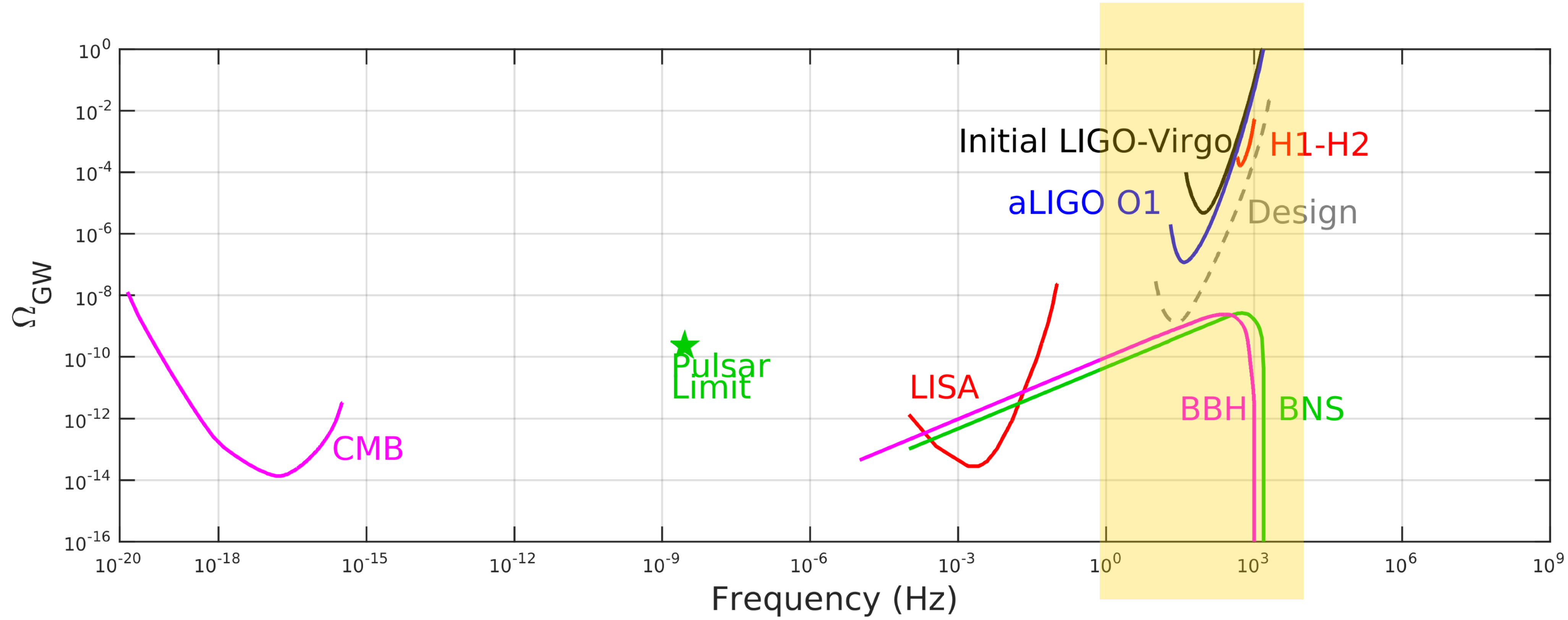
Expected future detection (4 yrs)

$$\Omega_{GW}(f) = \frac{f}{\rho_c}\frac{d\rho_{GW}(f)}{df} = \frac{f}{\rho_c}\int dz\,\frac{dE(z)}{df}\,R(z)\,\frac{n(z)}{H(z)}$$

density of sources

spectrum

rate of events



Abbott et al, PRL 118 (2017) 121101

upper bounds

$< 1.3 \cdot 10^{-9}$

$2.8 \times 10^{-9}\mathrm{Hz}$

Shannon et al., 2013

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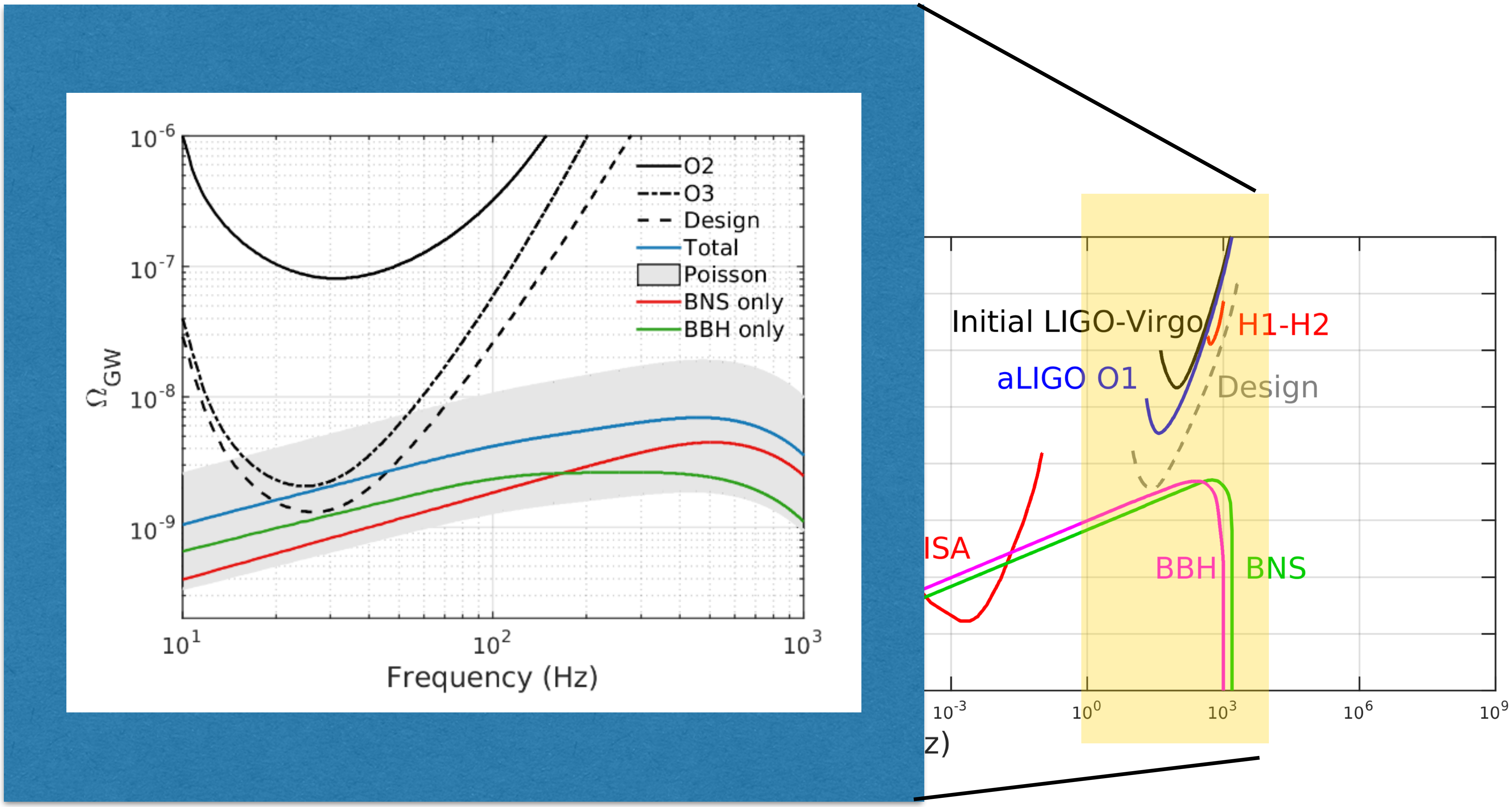
$25\mathrm{Hz}$

LIGO-Virgo
2019

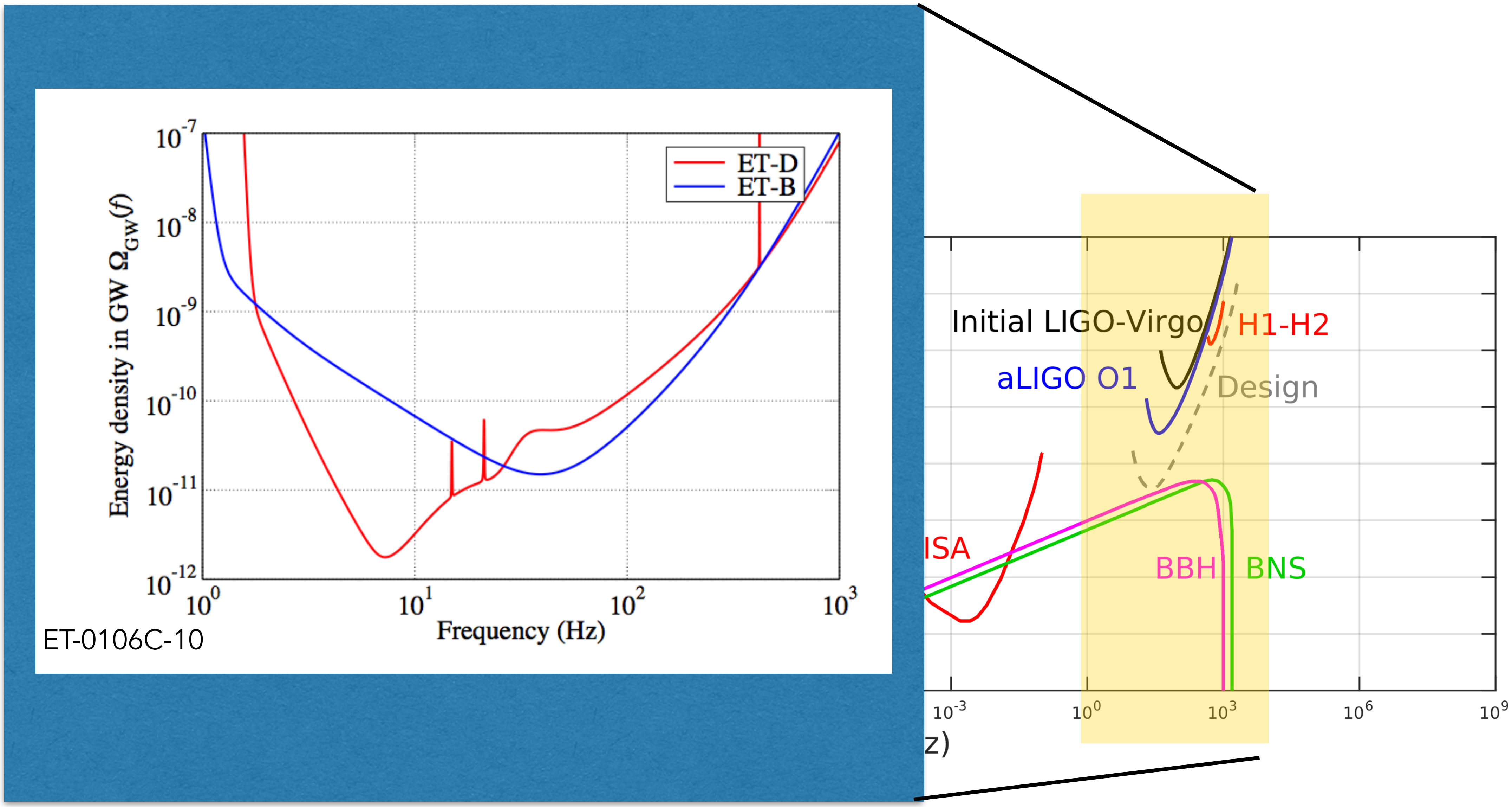
Expected future detection (4 yrs)



Abbott et al, PRL 118 (2017) 121101

Detection background very probable as the designed sensitivity is reached!

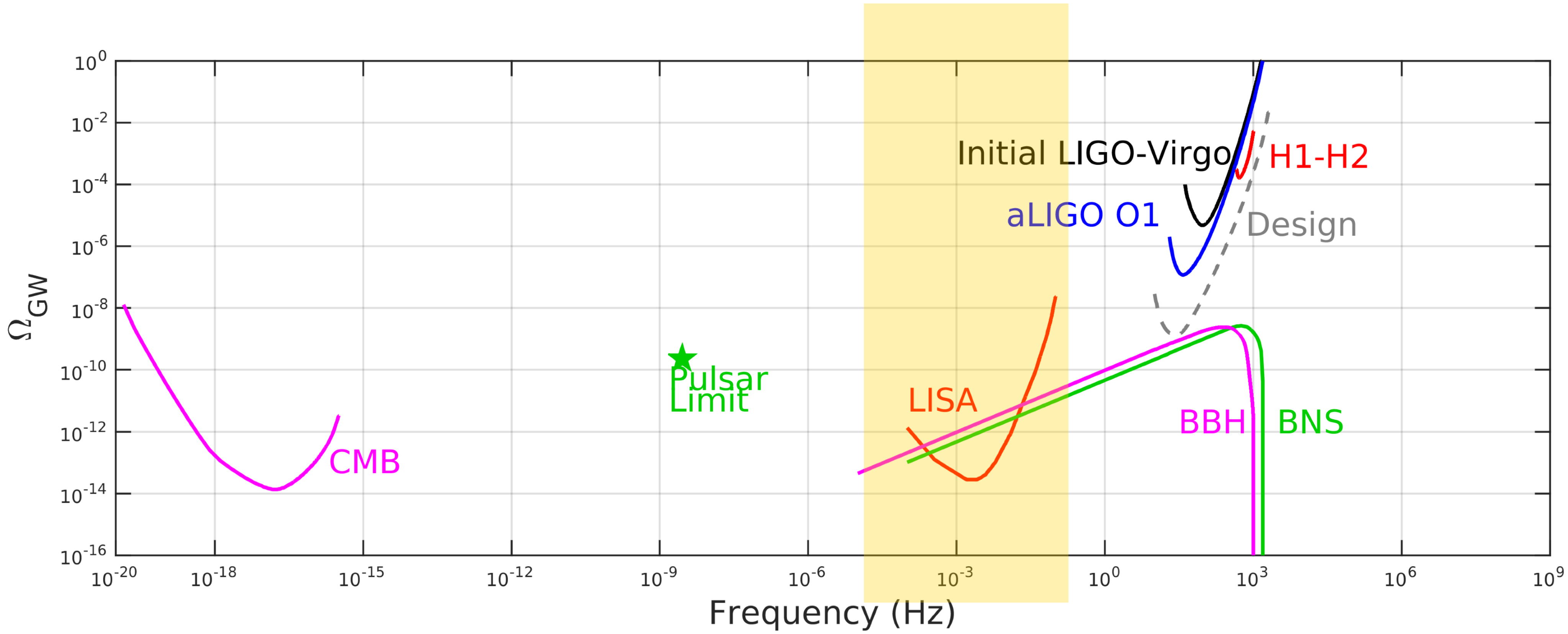
Einstein Telescope: improvement in sensitivity



ET-0106C-10

Improvement of a factor 10 in strain sensitivity: larger portion of the spectrum detected

LISA: inspiralling phase of mergers



LISA will see a continuous background (stationary) from inspiralling phase of binary systems

(vs LIGO will see merger phase of binary evolution. Popcorn-like signal)

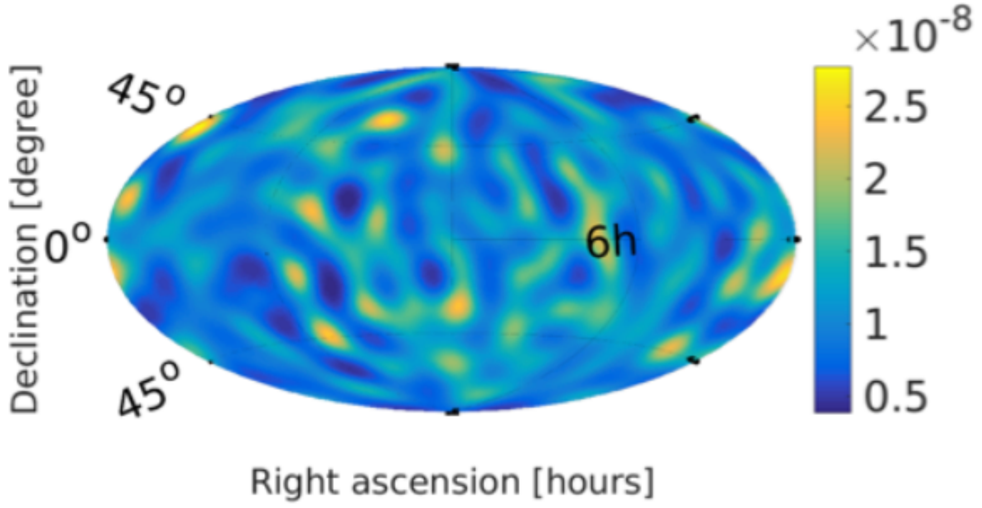
Angular searches: sky map

$$\Omega_{GW}(f) = \int d^2\mathbf{e}\,\boxed{\Omega_{GW}(f,\mathbf{e})}$$

$$\Omega_{GW}(f,\mathbf{e}) = \frac{f}{\rho_c}\,\frac{d^3\rho_{GW}(f,\mathbf{e})}{d^2\mathbf{e}\,df}$$

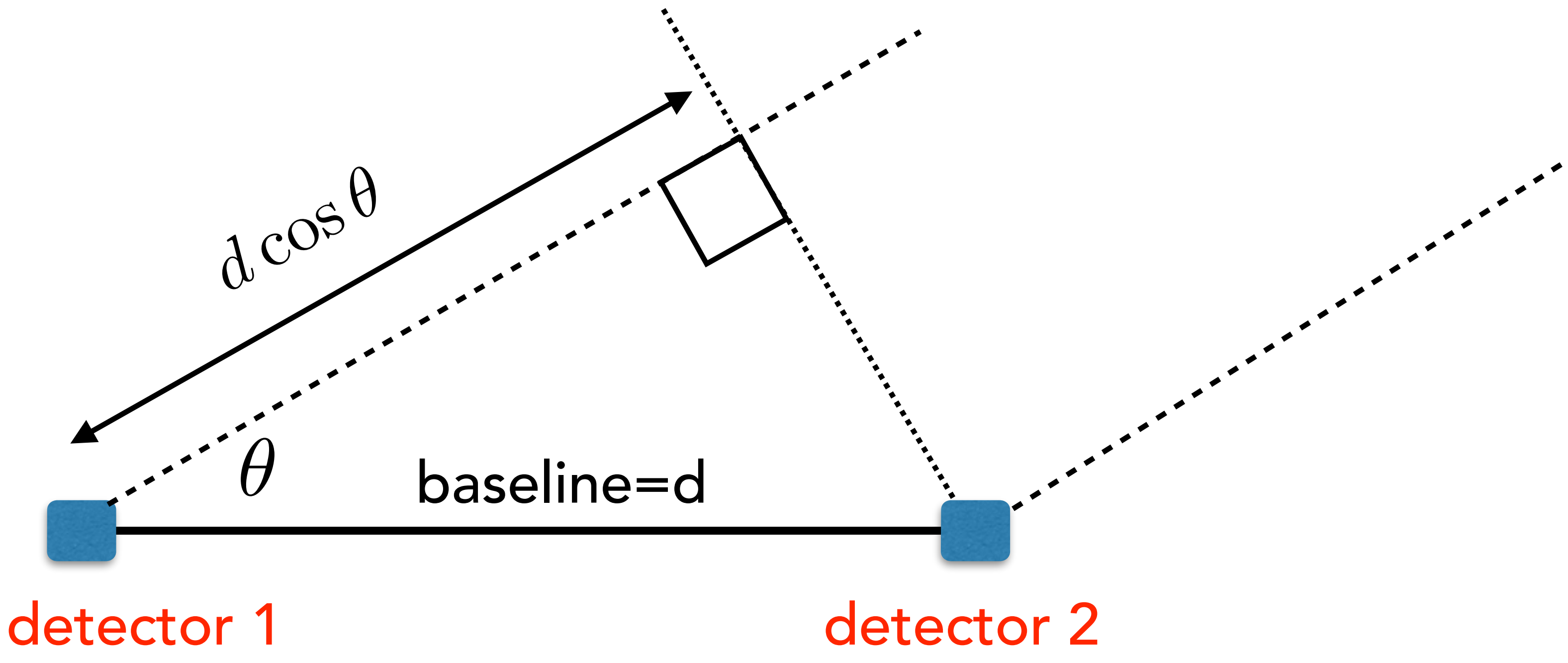
LIGO &Virgo : directional searches implemented: SNR consistent with gaussian noise

map of upper bounds



(LIGO-Virgo PRL 118, 121102, 2017)

Interferometric mapping



Angular resolution

$$\Delta\theta \sim \frac{\lambda}{d} \sim \frac{c}{fd}$$

$$\ell_{\max} = \frac{\pi}{\Delta\theta} \sim \frac{\pi f d}{c}$$

For LIGO detectors

$$\ell_{\max} = \frac{\pi f}{50\mathrm{Hz}} \sim 4$$

Adding Virgo —->>10

Standard assumption in searches

Frequency-direction factorization

$$\Omega_{GW}(f,\mathbf{e}) = \boxed{\left(\frac{f}{f_{\mathrm{ref}}}\right)^{\alpha}}\boxed{D(\mathbf{e})}$$

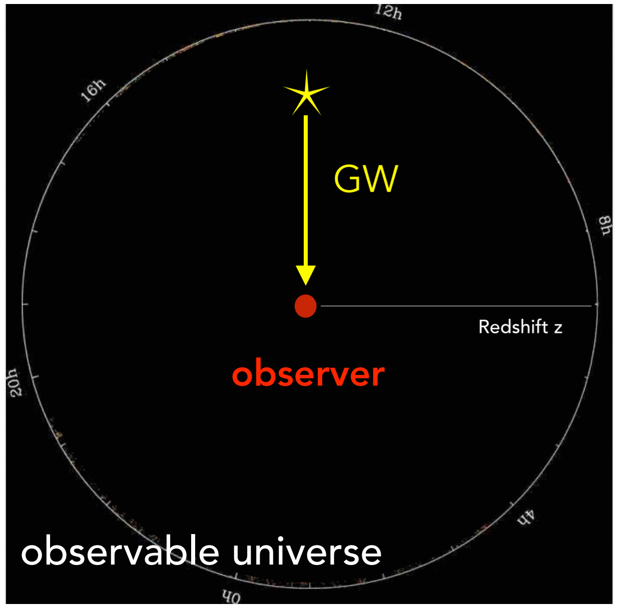
frequency
(power low)

direction

[we will test this assumption later with our framework]

Theoretical modeling and numerical predictions

Modeling side: standard description

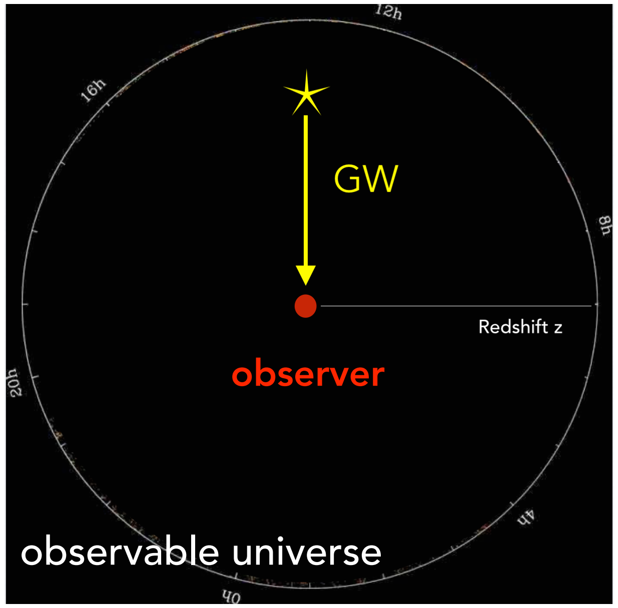


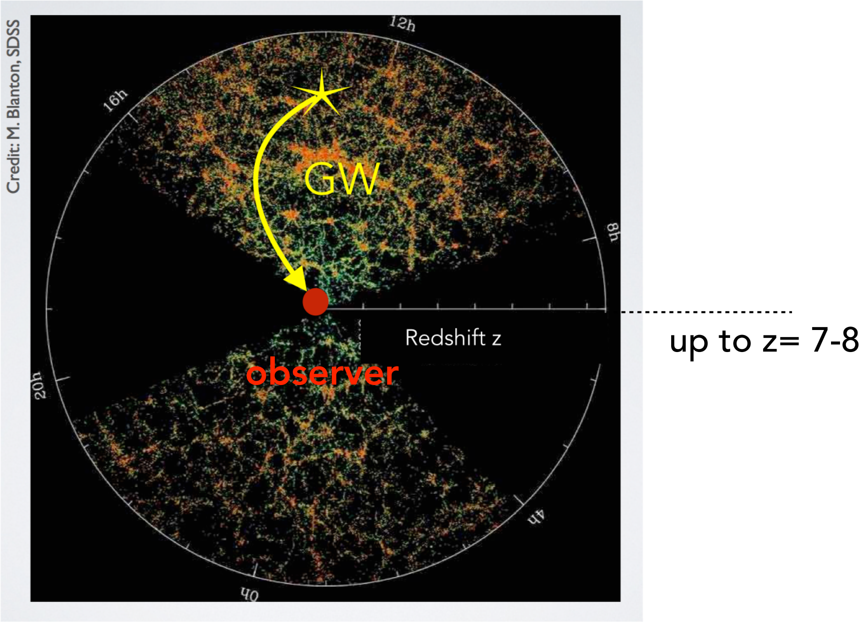
Usual modeling:
sources isotropically distributed,
propagation along straight line

—no anisotropies in the received flux

—no generation of polarization

Beyond the usual modeling: a realistic description





Usual modeling:
sources isotropically distributed,
propagation along straight line

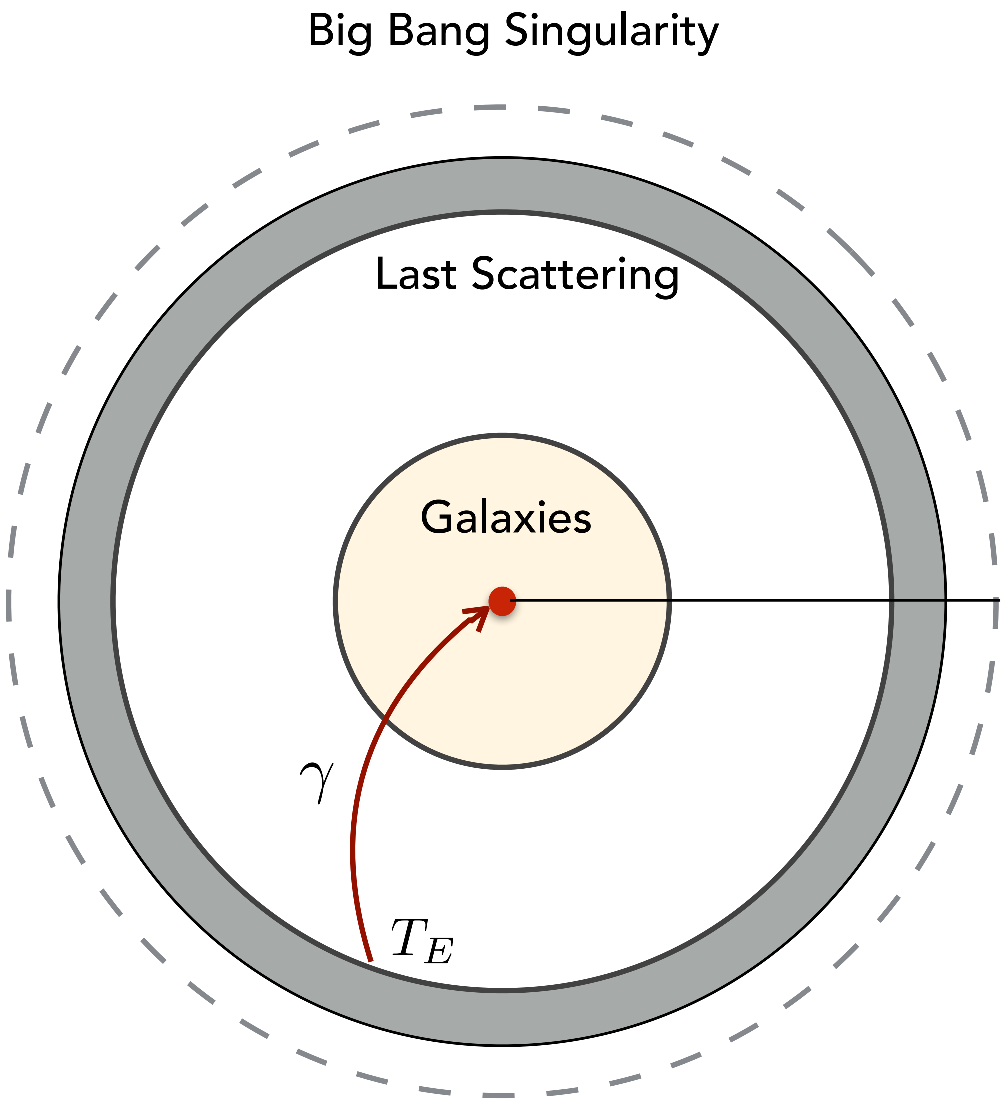
More realistic description:
including effects of inhomogeneities,
lensing, distortion

—no anisotropies in the received flux

—no generation of polarization

Accurate characterization of anisotropies and polarization

CMB: a case study



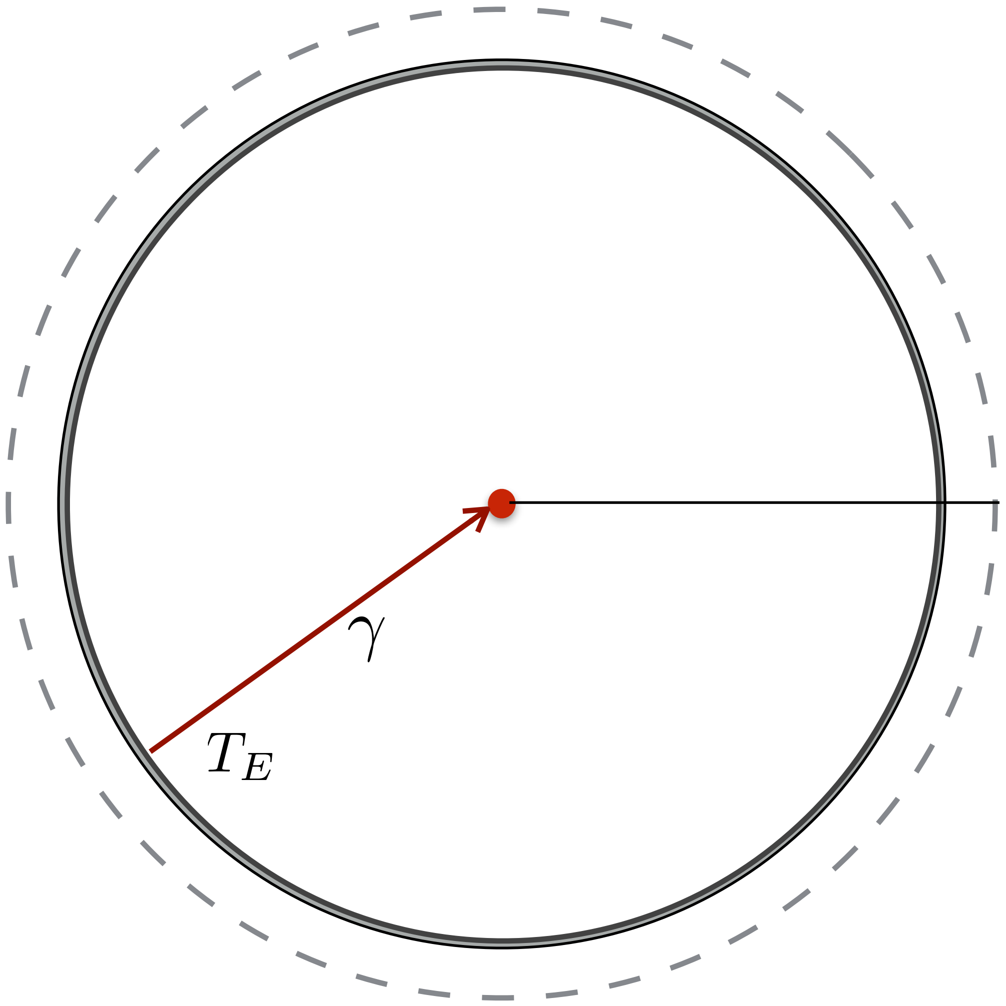
Temperature observed $\leftrightarrow$
temperature at Large Scattering Surface

$$\frac{T_O}{T_E} = \frac{E_O}{E_E} = \frac{(k^\mu u_\mu)_O}{(k^\mu u_\mu)_E}$$

wave vector

velocity comoving
observer

CMB assuming homogeneous and isotropic universe

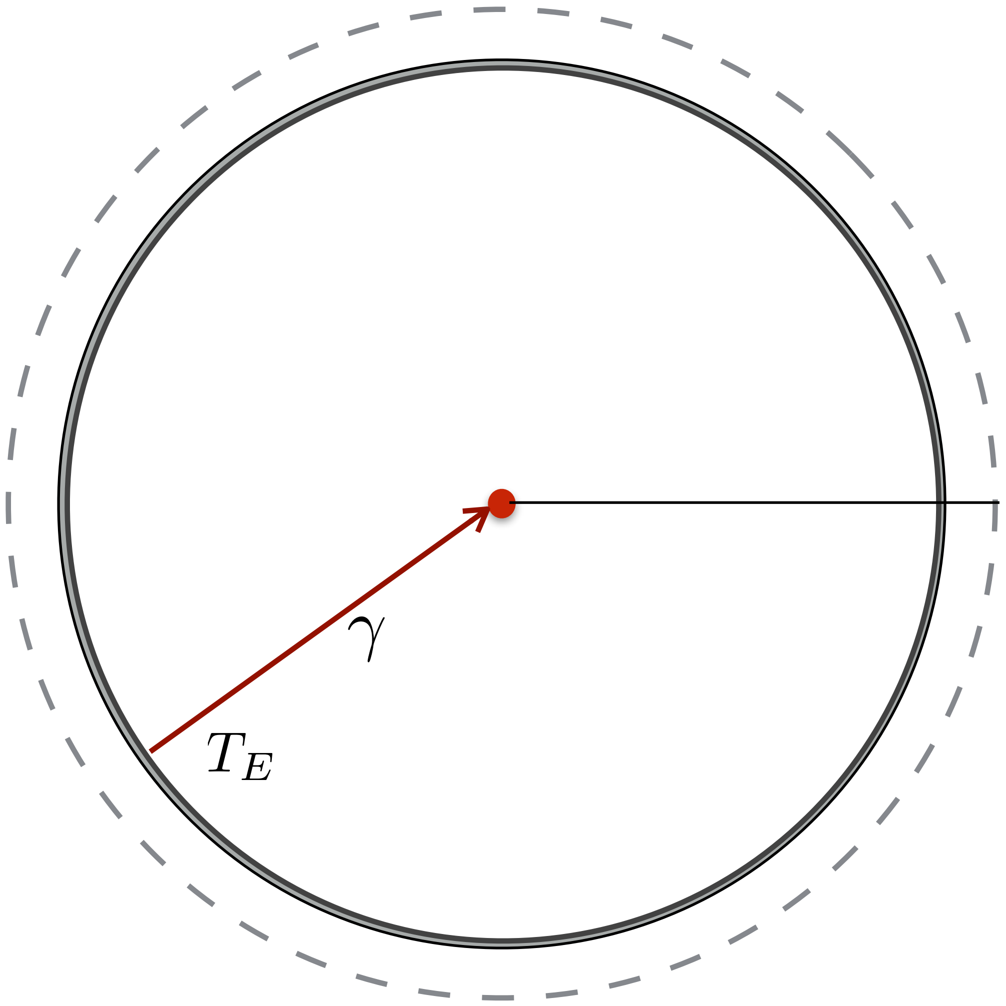


$$ds^2 = a^2(\eta)(-d\eta^2 + d\mathbf{x}^2)$$

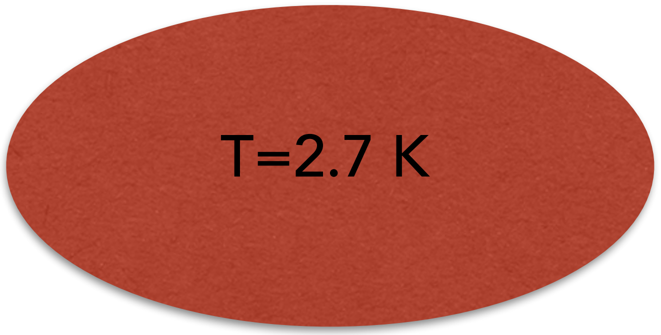
$$\frac{T_O}{T_E} = \frac{E_O}{E_E} = \frac{(k^\mu u_\mu)_O}{(k^\mu u_\mu)_E} = \frac{a(\eta_E)}{a(\eta_O)}$$

Temperature does not depend on directions!

CMB assuming homogeneous and isotropic universe

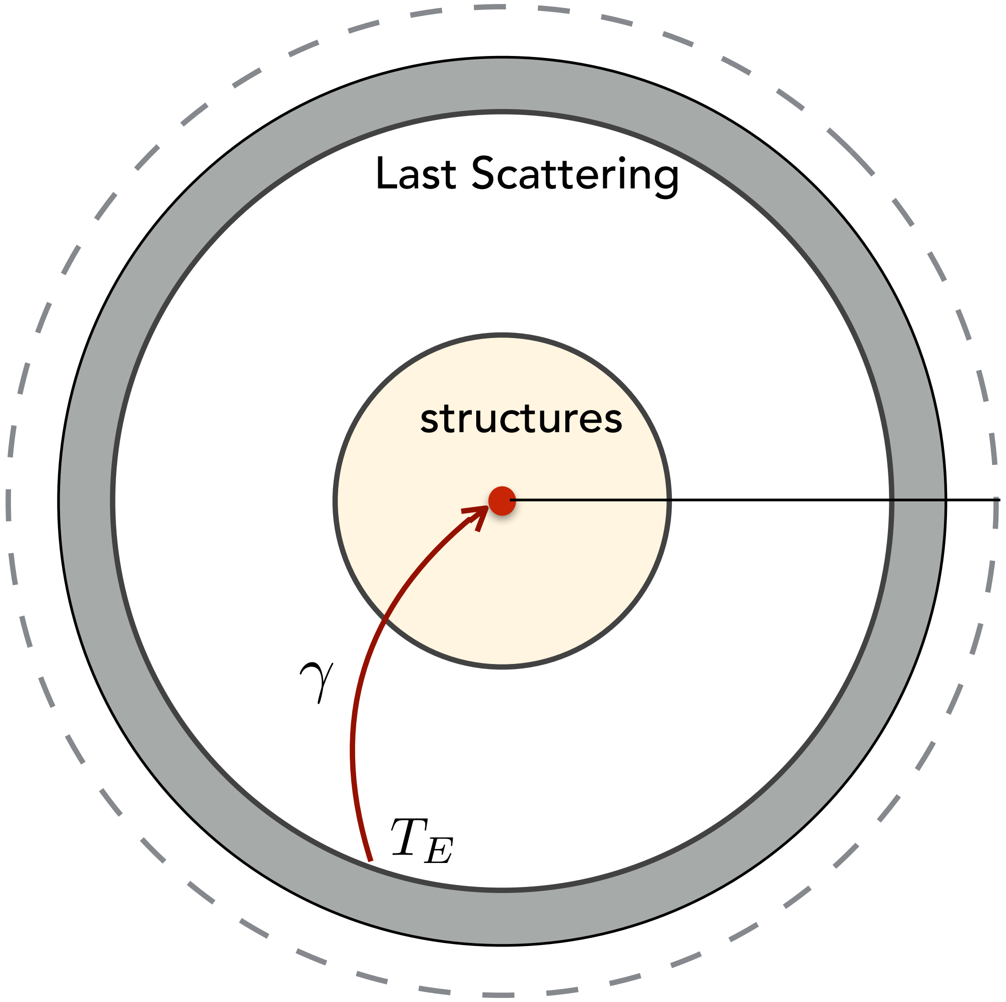


sky map



(see Penzias & Wilson ’65)

CMB: effects of structures

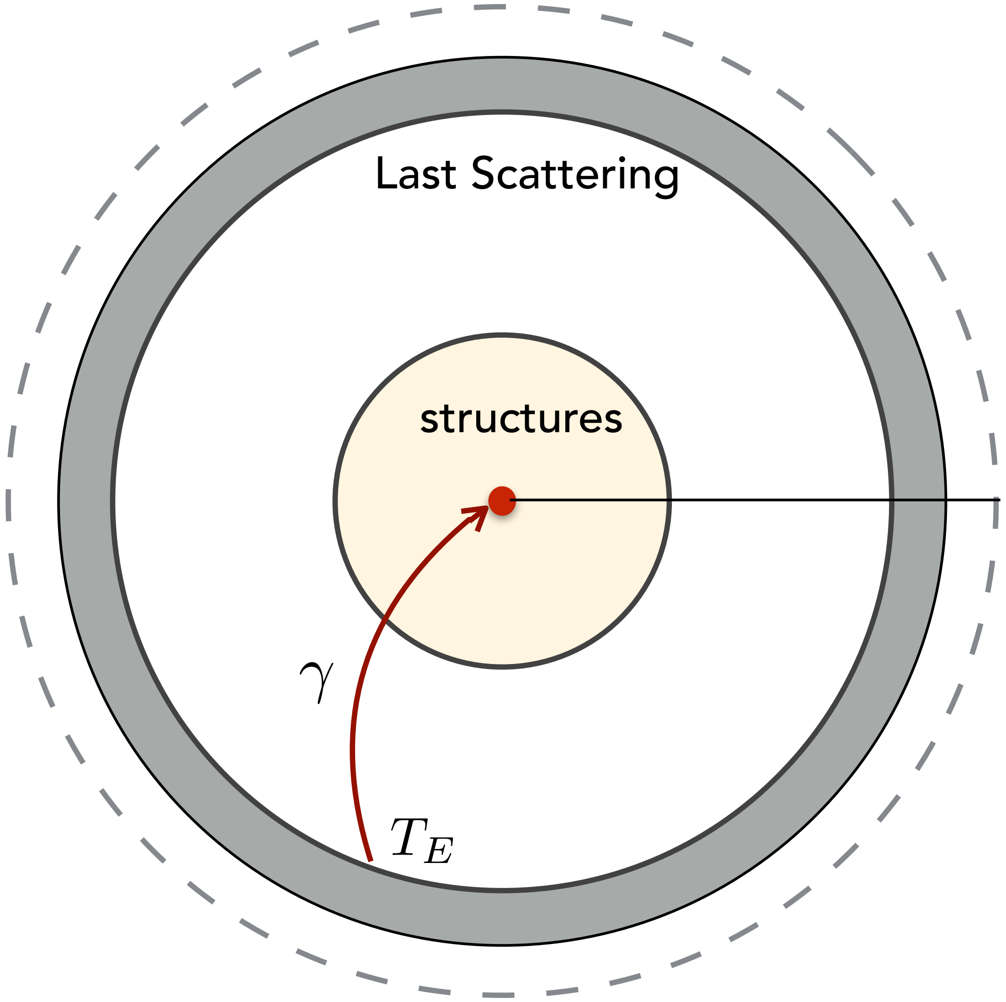


$$ds^2 = a^2\left[-(1+2\Psi)d\eta^2 + (1-2\Phi)\delta_{ij}dx^i dx^j\right]$$

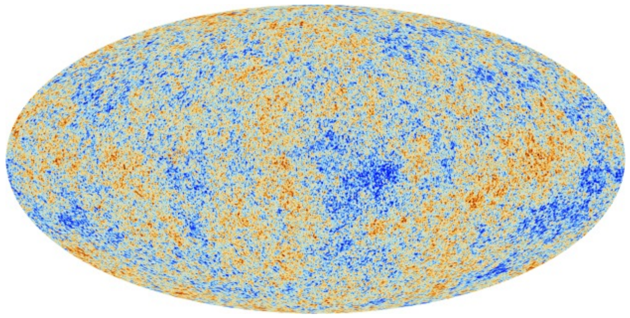
$$T_O(\mathbf{e}) = \bar{T}_O(\eta_O)(1+\Theta(\mathbf{e}))$$

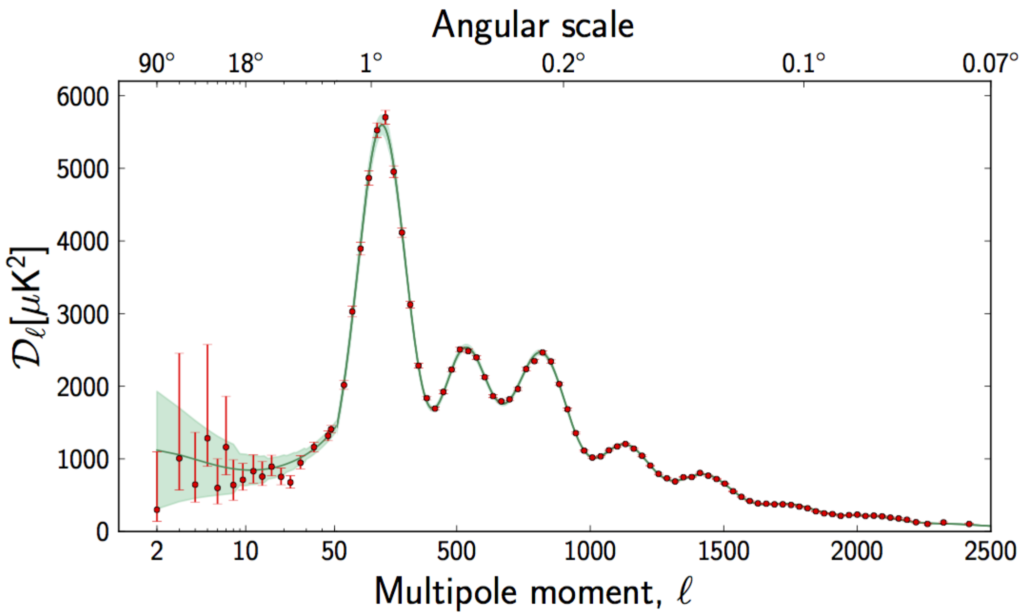
$$\Theta(\mathbf{e},\mathbf{x}_O,\eta_O) = \left(\frac{1}{4}\delta_\gamma + \Phi - \mathbf{e}\cdot\mathbf{v}\right)(\mathbf{x}_E,\bar{\eta}_E) + \int_E^O (\Phi' + \Psi')d\eta$$ (Sachs-Wolfe formula '67)

CMB: constraining cosmology



sky map

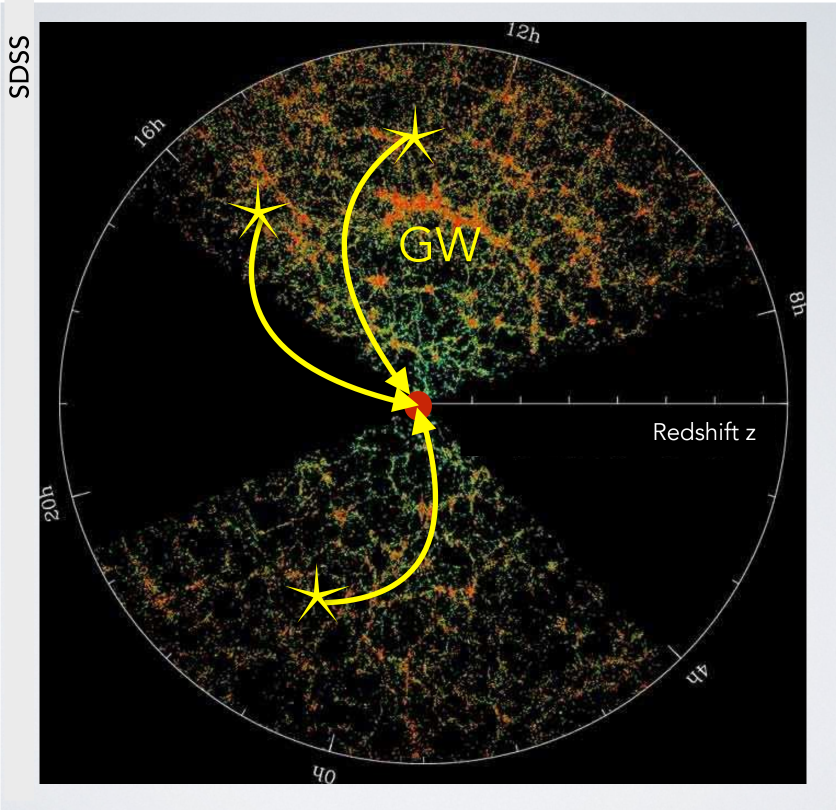




$$\Theta(\mathbf{e}, \mathbf{x}_O, \eta_O) \longrightarrow$$

GW background case

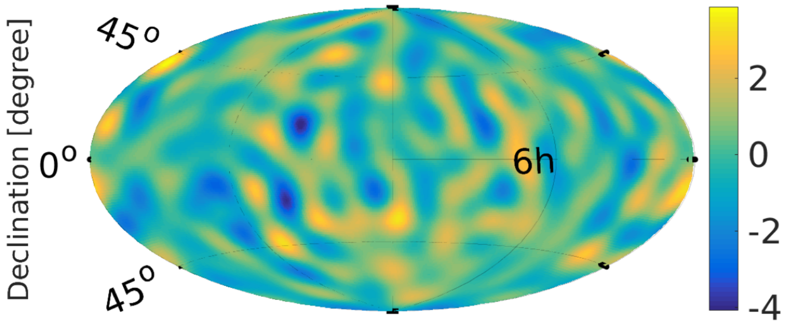
Predictions

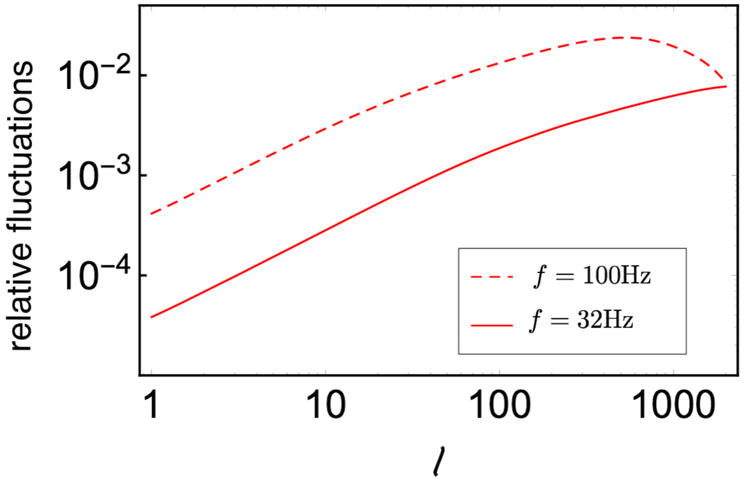


$$\Omega_{GW}(f, \mathbf{e}) \longrightarrow$$

sky map (future)

(LIGO-VIRGO PRL 118, 121102, 2017)

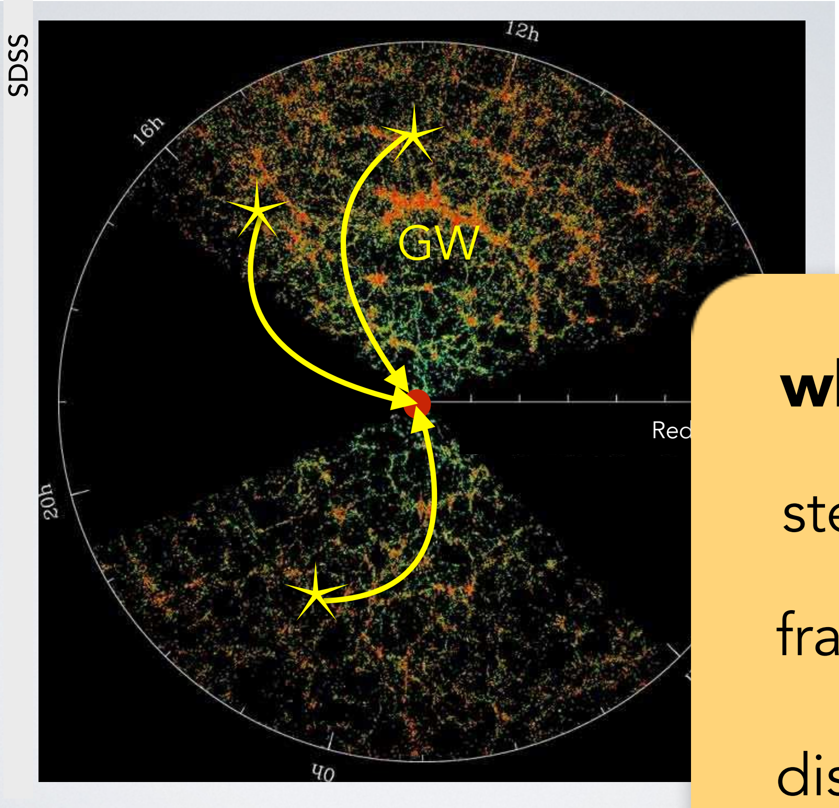




[Cusin, Dvorkin, Uzan, Pitrou 2018]

Constrain astrophysical functions and parameters

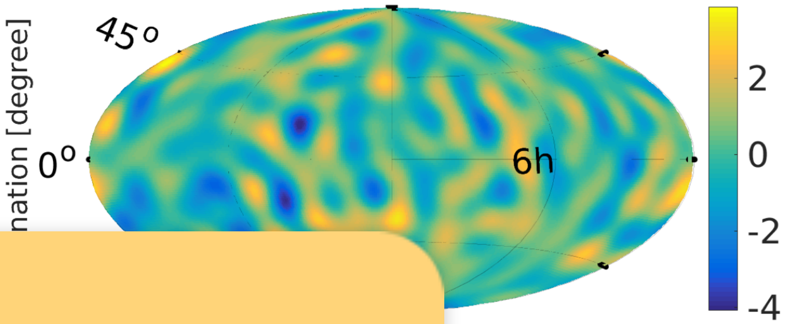
Predictions



$$\Omega_{GW}(f, \mathbf{e})$$

sky map (future)

(LIGO-VIRGO PRL 118, 121102, 2017)



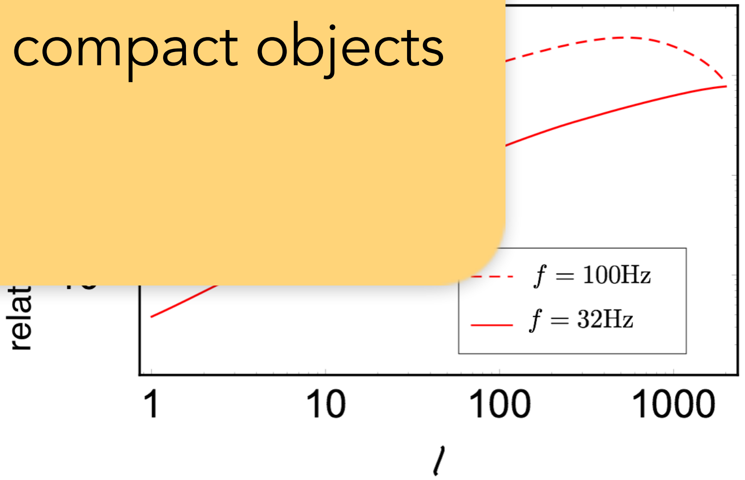
what we can learn

stellar evolution model

fraction black holes in binary systems

distribution masses of compact objects

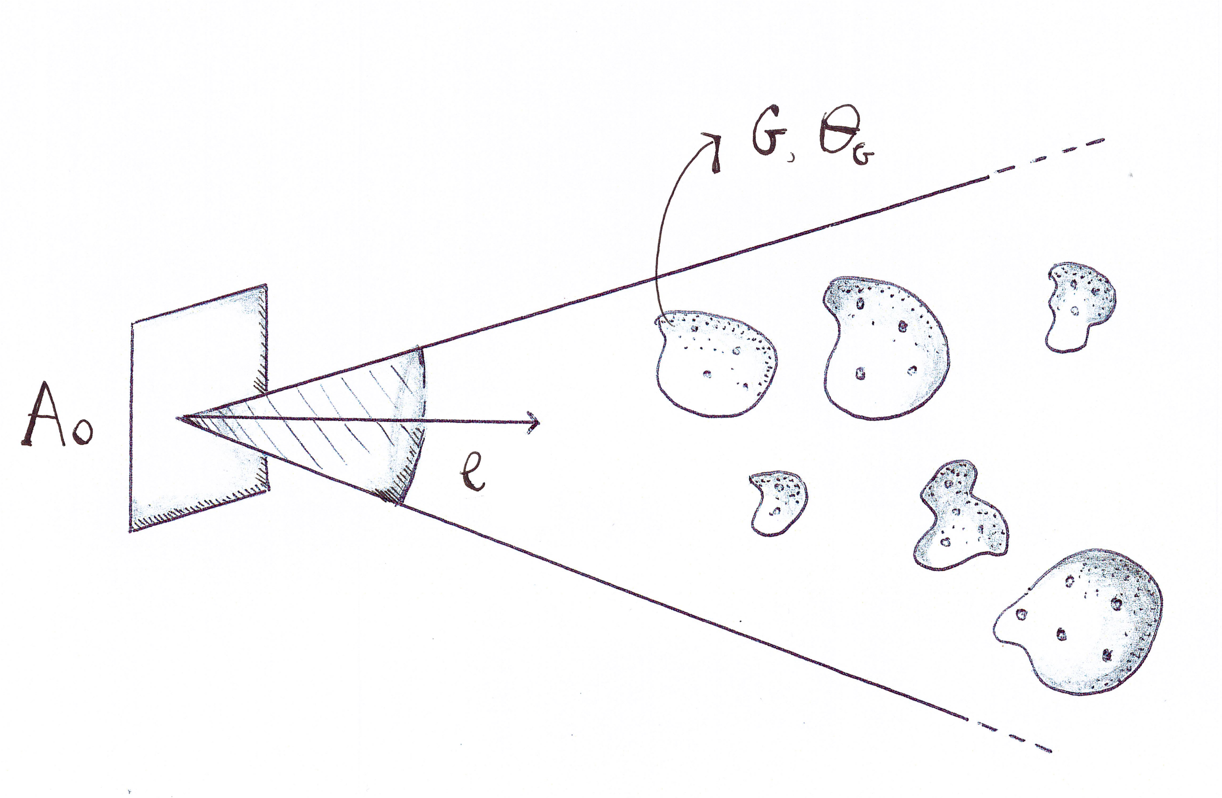
…



[Cusin, Dvorkin, Uzan, Pitrou 2018]

Scheme of our approach

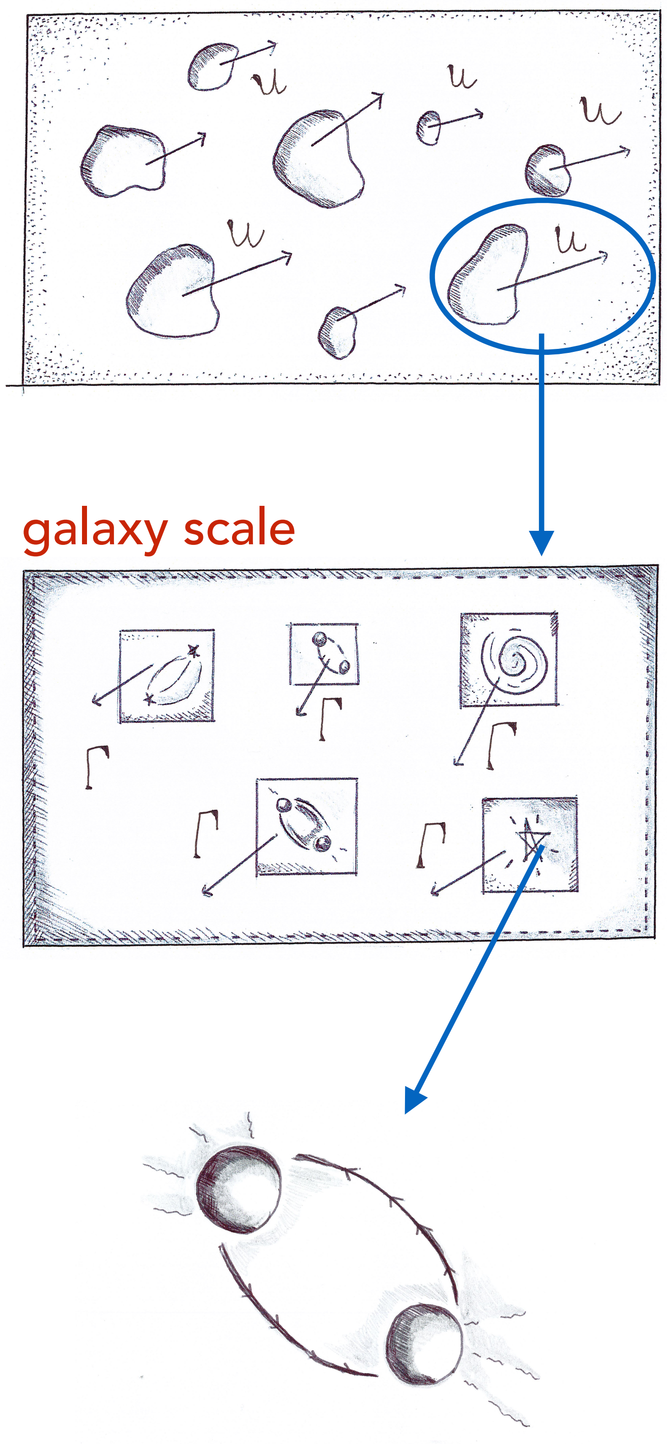
Observer looks at the sky in a given direction



$$\Phi(\mathbf{e}, z_G, \theta_G) = \frac{\text{Energy}}{A_O \Delta t_O}$$

Total flux received: sum the contributions from all the galaxies in the solid angle of observation

Three scales in the problem

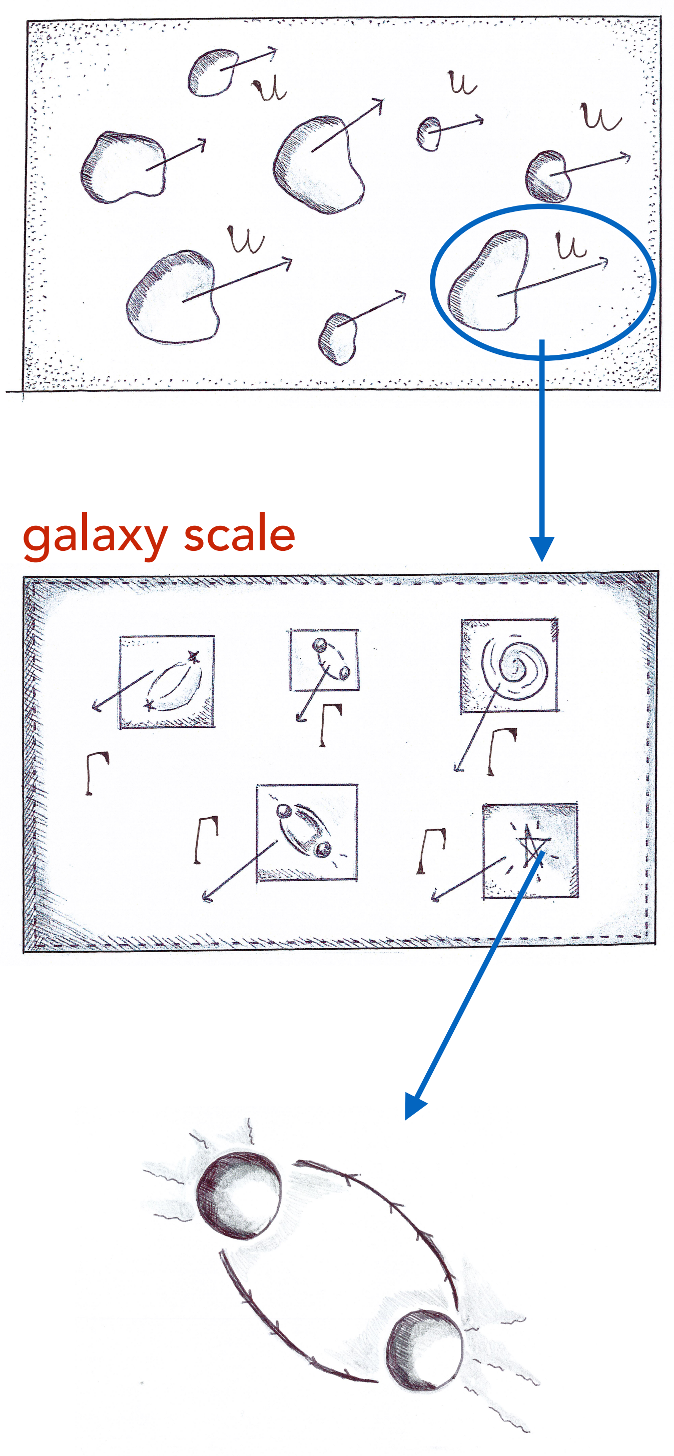


cosmological scale. Galaxies: point-like sources moving with the cosmic flow

galactic scale. Effective luminosity of a galaxy defined taking into account the various contributions of the sources

local scale: single GW sources inside a galaxy

From cosmological to local scale



cosmological scale

local scale

$$\Phi = \frac{(1+z_G)}{D_L^2}\boxed{\mathcal{L}_G}$$

function local
quantities at sources

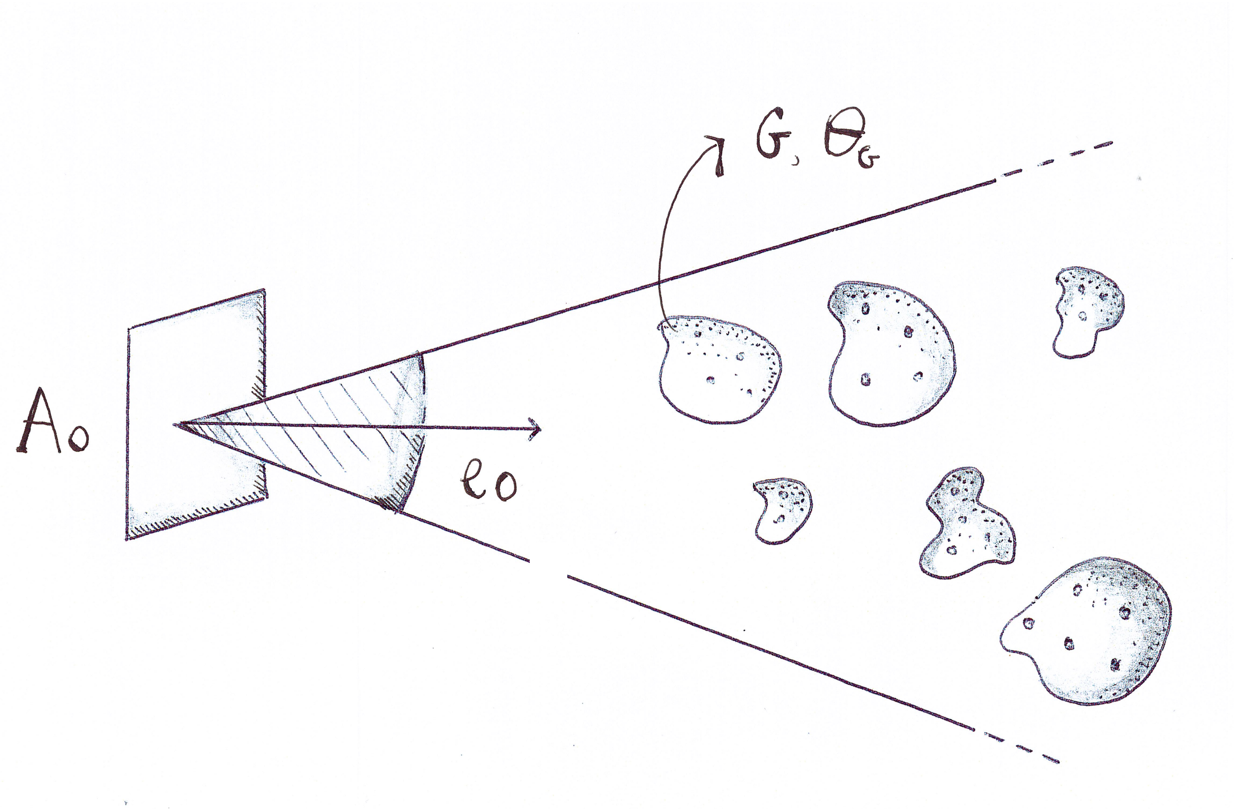
Final parametrization

integral over halo mass function

$$\Omega_{GW}(f,\mathbf{e}) = \frac{f}{\rho_c}\int dz_G \int d\theta_G \, \boxed{\Phi[z_G,f,\theta_G]}\,\boxed{\frac{d^3\mathcal{N}_G}{dz_G\, d^2\mathbf{e}}(z_G,\theta_G)}$$

flux from one galaxy

galaxies in comoving volume



GC, Pitrou, Uzan, Phys.Rev. D96 (2017)

Final parametrization

$$\Omega_{GW}(f,\mathbf{e}) = \frac{f}{\rho_c}\int dz_G \int d\theta_G \overset{\text{astrophysical component}}{\boxed{\Phi[z_G, f, \theta_G]}}\overset{\text{cosmological component}}{\boxed{\frac{d^3\mathcal{N}_G}{dz_G d^2\mathbf{e}}(z_G,\theta_G)}}$$

rewritten in terms of comoving density and comoving volume

rewritten in terms of luminosity

GC, Pitrou, Uzan, Phys.Rev. D96 (2017)

Results in cosmological context

astrophysical part cosmological part

$$\delta\Omega_{GW}(\mathbf{e},f)=\frac{f}{4\pi\rho_c}\int_{\eta_*}^{\eta_O}\mathrm{d}\eta\,\mathcal{A}(\eta,f)\left[b\delta_m+4\Psi-2\mathbf{e}\cdot\nabla v+6\int_{\eta}^{\eta_O}\mathrm{d}\eta'\dot{\Psi}\right]$$

$$\mathcal{A}(\eta,f)\equiv a^4\bar{n}_{\rm G}(\eta)\int d\theta_{\rm G}\mathcal{L}_{\rm G}(\eta,f_{\rm G},\theta_{\rm G})$$

$$\Omega_{GW}(f,\mathbf{e})=\bar{\Omega}_{GW}(f)+\boxed{\delta\Omega_{GW}(f,\mathbf{e})}$$

Astrophysical model: ingredients

(1)
$$\mathcal{L}_{\rm G} = \int \mathrm{d}m\,\mathrm{d}m'\,\mathrm{d}a_{\rm f}\,\boxed{\frac{\mathrm{d}E}{\mathrm{d}f}} \times \boxed{\mathcal{R}_{\rm m}[m, m', a_{\rm f}, t]}$$

spectrum

merger rate

star formation rate

stellar evolution model

fraction of black holes in binary systems

mass range of compact objects …

(2) **sum over galaxy population** using the halo mass function calibrated with simulations

GC, Dvorkin, Pitrou, Uzan PRL 120 (2018) 231101

Dvorkin, Uzan, Vangioni, Silk, Phys.Rev.D94 (2016), 103011

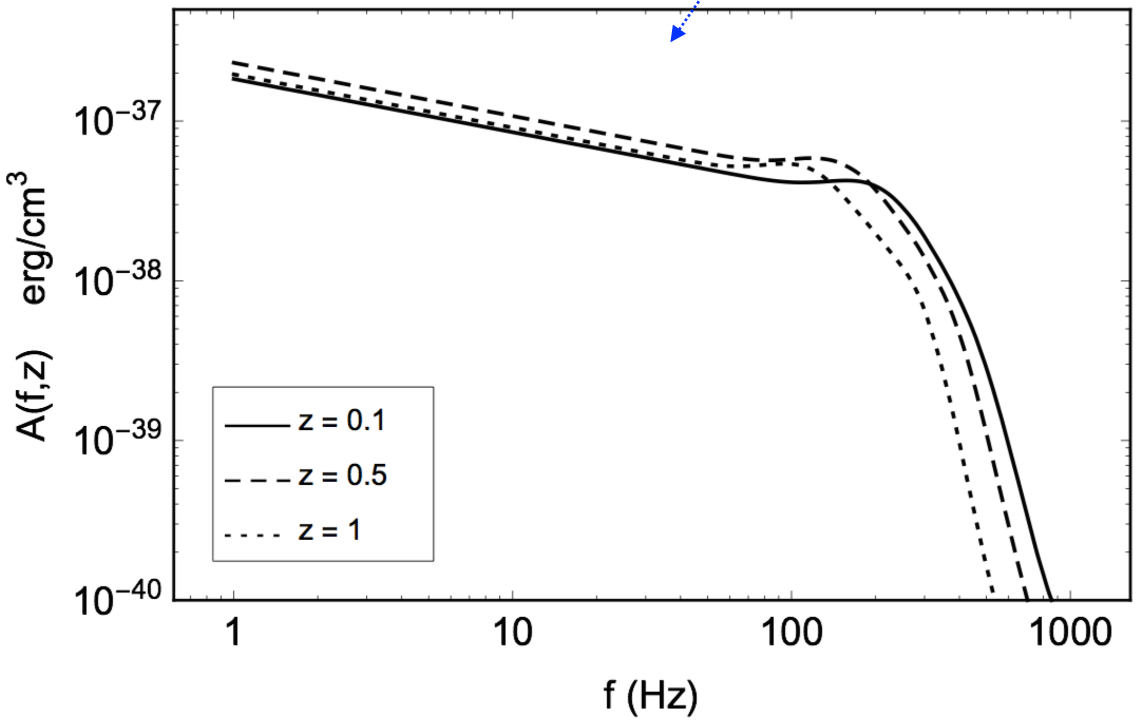
Astrophysical kernel for a reference model

astrophysical part

cosmological part

$$\delta\Omega_{GW}(\mathbf{e},f) = \frac{f}{4\pi\rho_c}\int_{\eta_*}^{\eta_O} \mathrm{d}\eta\, \mathcal{A}(\eta,f)\left[b\delta_m + 4\Psi - 2\mathbf{e}\cdot\nabla v + 6\int_{\eta}^{\eta_O}\mathrm{d}\eta'\dot{\Psi}\right]$$

$$\mathcal{A}(\eta,f) \equiv a^4\bar{n}_{\mathrm{G}}(\eta)\int d\theta_{\mathrm{G}}\mathcal{L}_{\mathrm{G}}(\eta,f_{\mathrm{G}},\theta_{\mathrm{G}})\,,$$



GC, Dvorkin, Pitrou, Uzan, 2018

Angular power spectrum

Non-vanishing auto-correlation linked to **correlation of large scale structures** (with modulation from local physics)

$$C(f,\theta) = \langle \delta\Omega_{GW}(f,\mathbf{e}_1)\delta\Omega_{GW}(f,\mathbf{e}_2)\rangle$$

$$\sum_\ell \frac{2\ell+1}{2\pi} C_\ell(f) P_\ell(\mathbf{e}_1\cdot\mathbf{e}_2)$$

depends on frequency

$$\boxed{C_\ell(f) = \frac{2}{\pi}\int \mathrm{d}k\, k^2 |\delta\hat{\Omega}_\ell(k,f)|^2}$$

Correlation with other cosmological observables

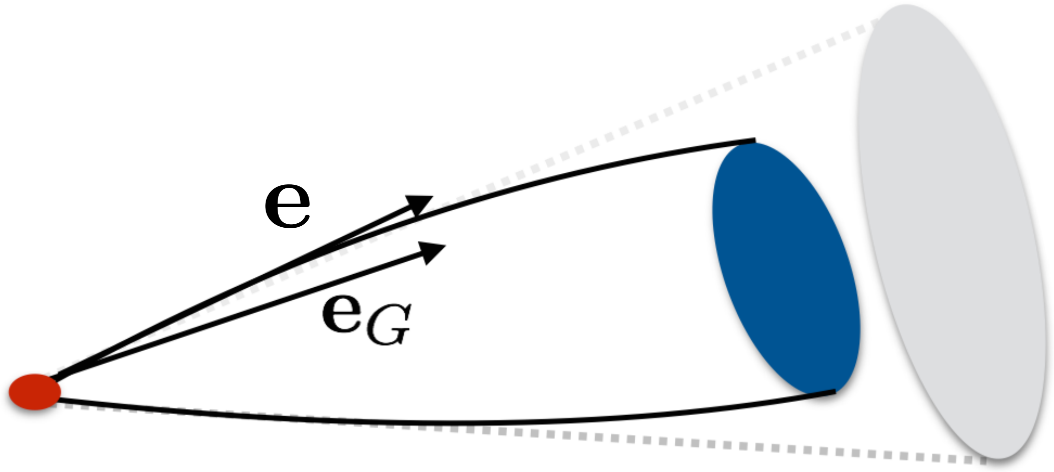
Number counts: number of galaxies as a function of direction and redshift

(see e.g. Bonvin & Durrer 2011)

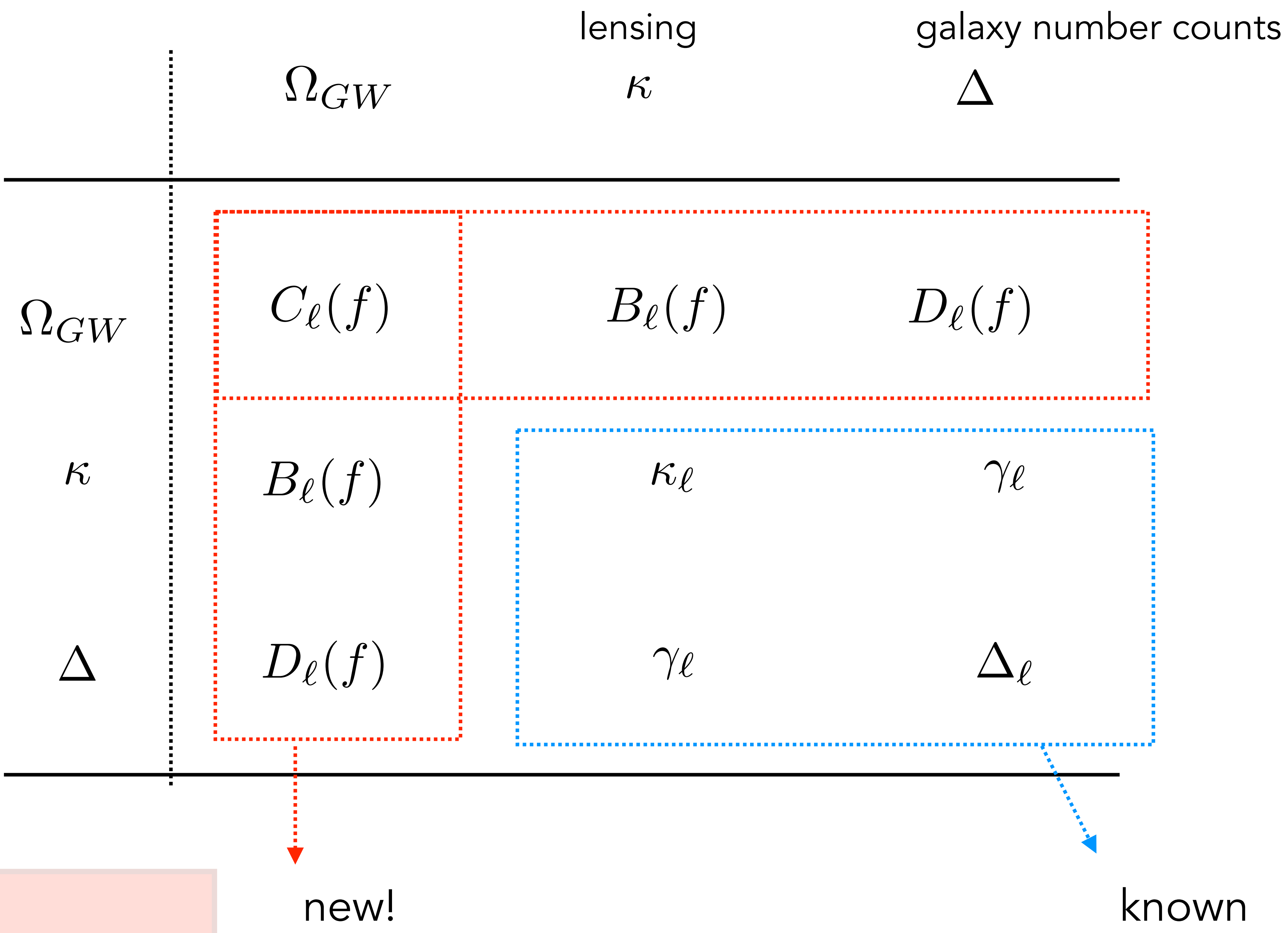
Weak lensing describes the deformation of the shape of a given galaxy by the gravitational potential of the large scale structures

$$\mathbf{e}_G = \mathcal{A} \cdot \mathbf{e}$$

amplification matrix



Summary of correlations



		lensing	galaxy number counts
	Ω_{GW}	κ	Δ
Ω_{GW}	$C_\ell(f)$	$B_\ell(f)$	$D_\ell(f)$
κ	$B_\ell(f)$	κ_ℓ	γ_ℓ
Δ	$D_\ell(f)$	γ_ℓ	Δ_ℓ

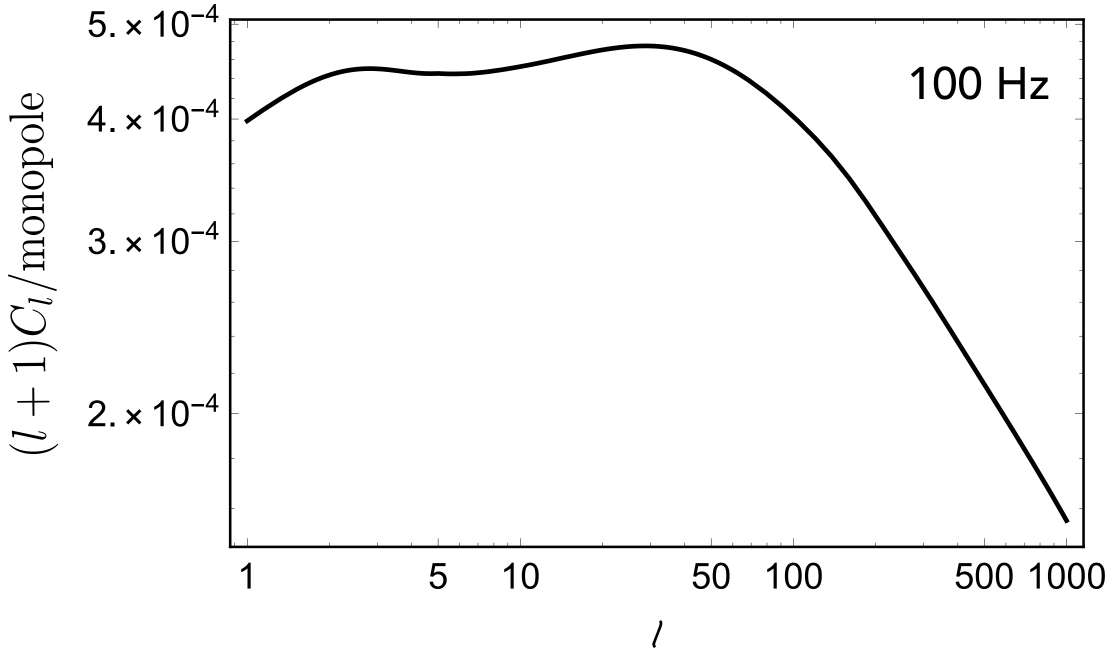
new!

known

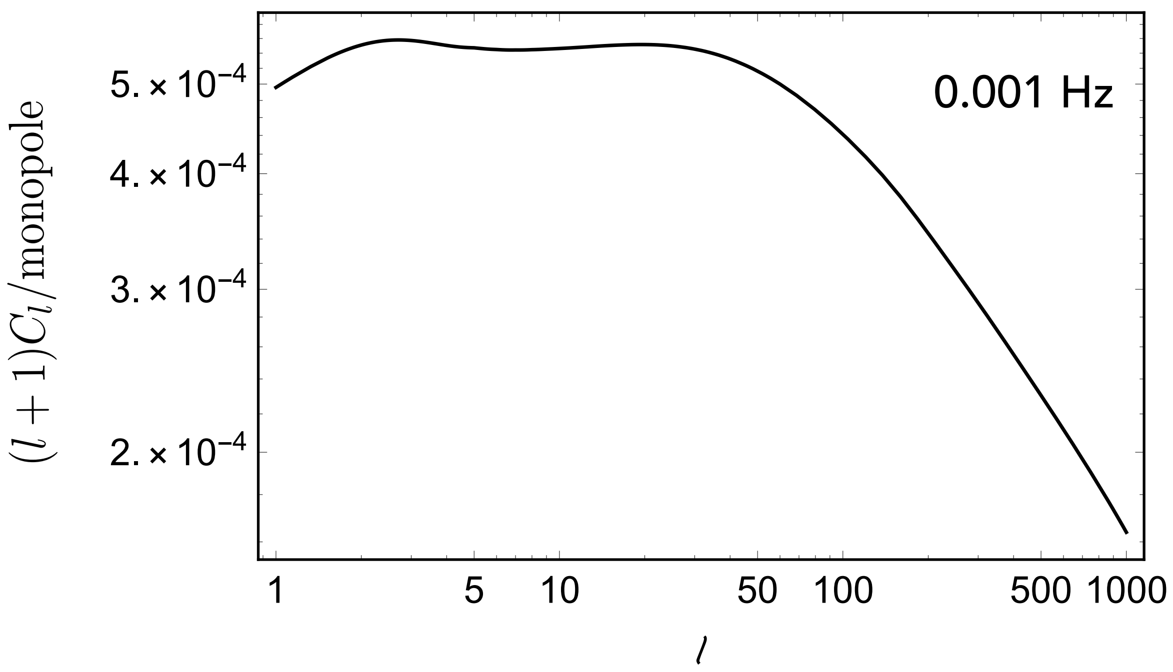
GC, Pitrou, Uzan,
Phys.Rev. D96 (2017) 10, 103019

Angular power spectrum

LIGO-Virgo band



LISA band



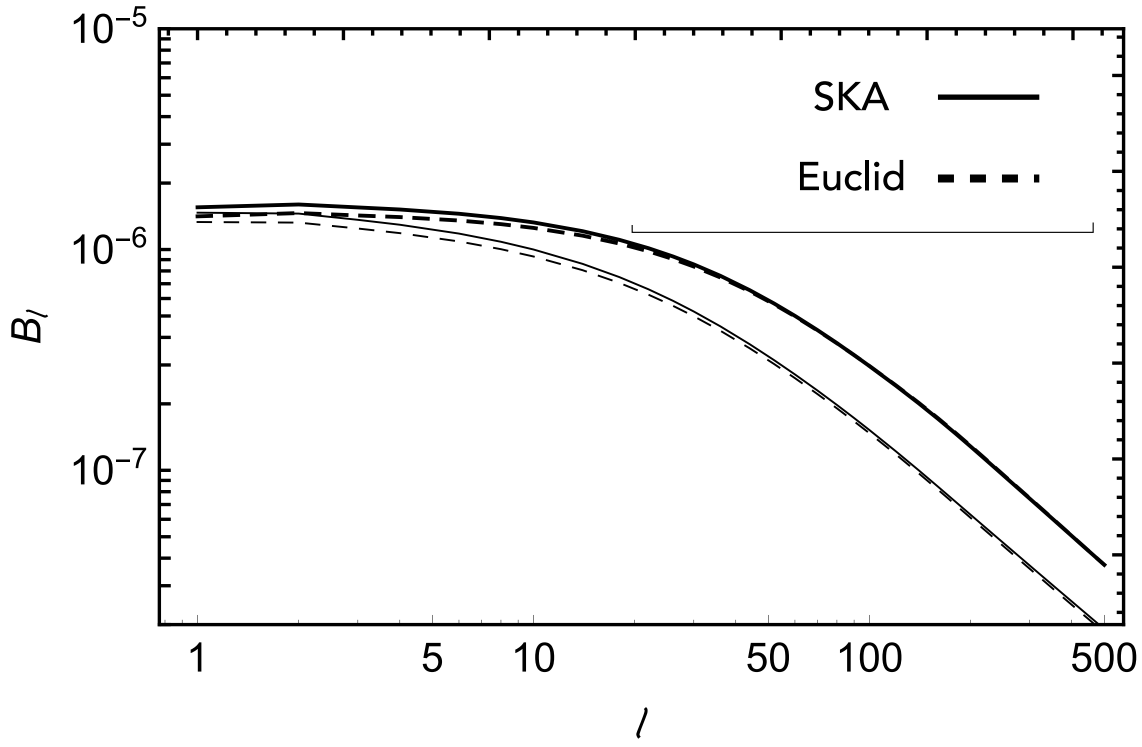
detectable by Einstein Telescope/Cosmic Explorer (?)

$$\left(\ell+\frac{1}{2}\right)C_\ell(f) \simeq \int dk\, P_\delta(k)$$ for large angular scales

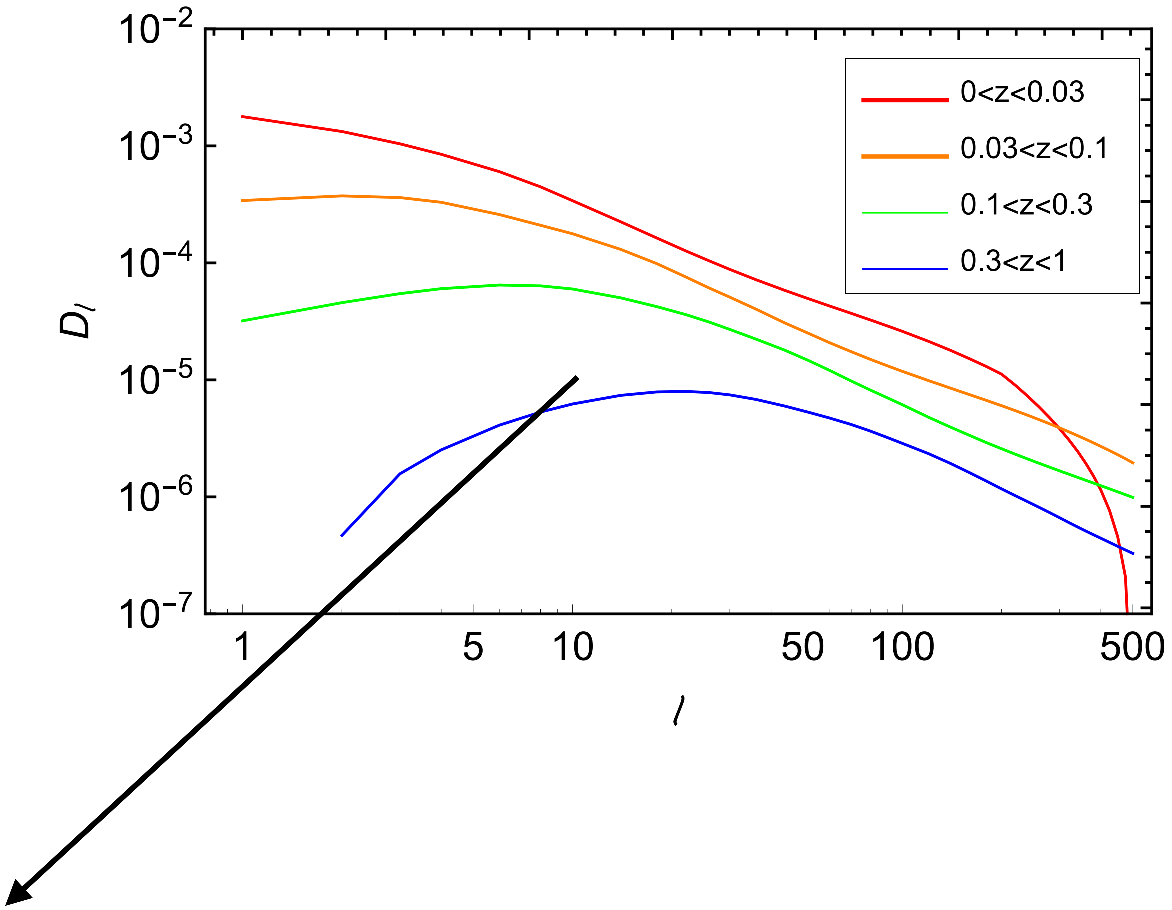
GC, Dvorkin et al. PRL 120 (2018) 231101
GC, Dvorkin et al. MNRAS Lett (2019)

Cross-correlations

cross correlation with weak lensing



cross correlation with galaxy number counts



window function at different redshifts

allows tomographic reconstruction: contribution from different redshifts

allows distinguish astrophysical GW background from cosmological one

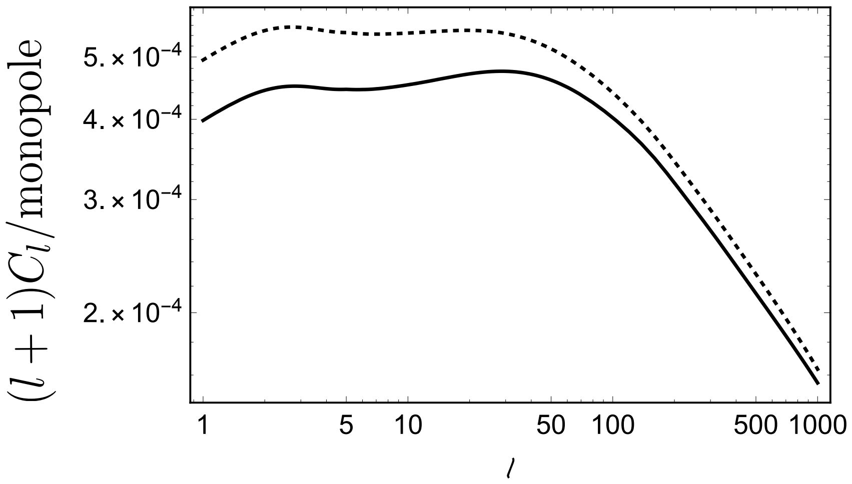
Relevance of this study for astrophysics

$$\boxed{\delta\Omega_{GW}(\mathbf{e},f)} = \frac{f}{4\pi\rho_c}\int_{\eta_*}^{\eta_O} \mathrm{d}\eta\, \overset{\text{astrophysical part}}{\mathcal{A}(\eta,f)} \overset{\text{cosmological part}}{\left[b\delta_m + 4\Psi - 2\mathbf{e}\cdot\nabla v + 6\int_{\eta}^{\eta_O} \mathrm{d}\eta'\dot{\Psi}\right]}$$

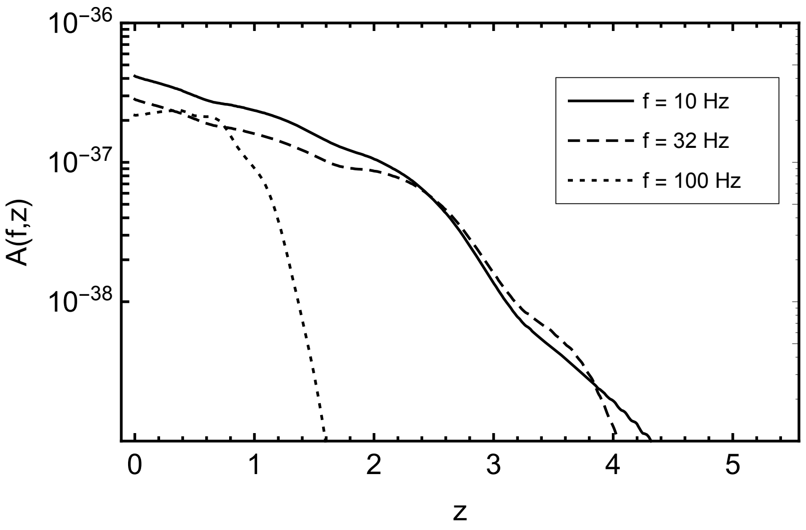
anisotropies per units of frequency and directions

- stellar evolution model
- fraction compact objects in binaries
- mass range
- distribution parameters binary system…

- metric perturbations
- peculiar velocities
- matter over density
- galaxy bias …



Astro kernel for a reference model



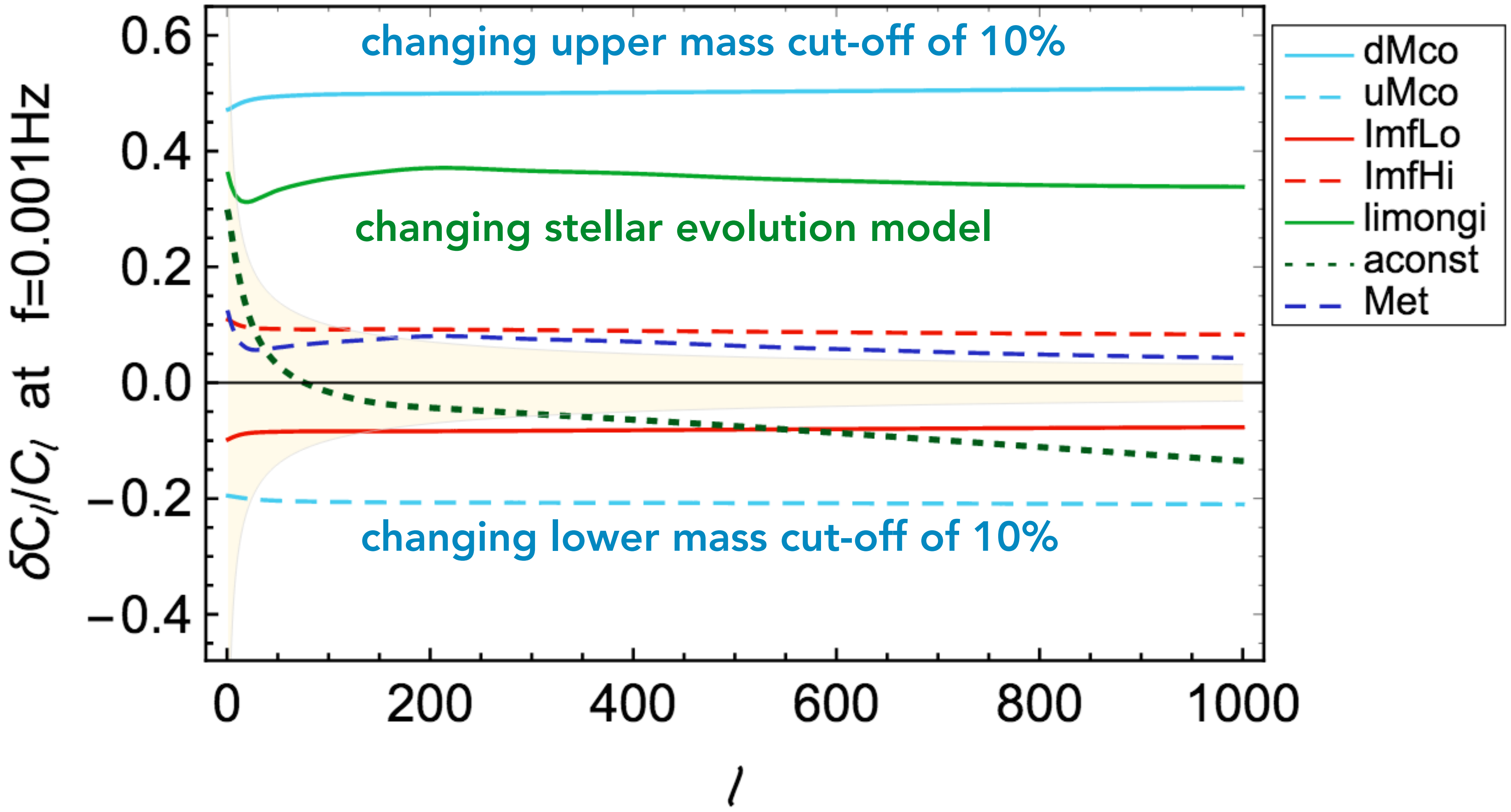
cosmological transfer functions for perturbations

+

$$T(\eta)$$

Test astrophysical modeling: explorative approach

Fractional differences with respect to a reference model



variation >40%

GC, Dvorkin et al. Phys.Rev. D100 (2019) 063004
GC, Dvorkin et al. MNRAS Lett (2019)

Ongoing work: how to extract astro info

What we know: there is astro info in anisotropies of GW background

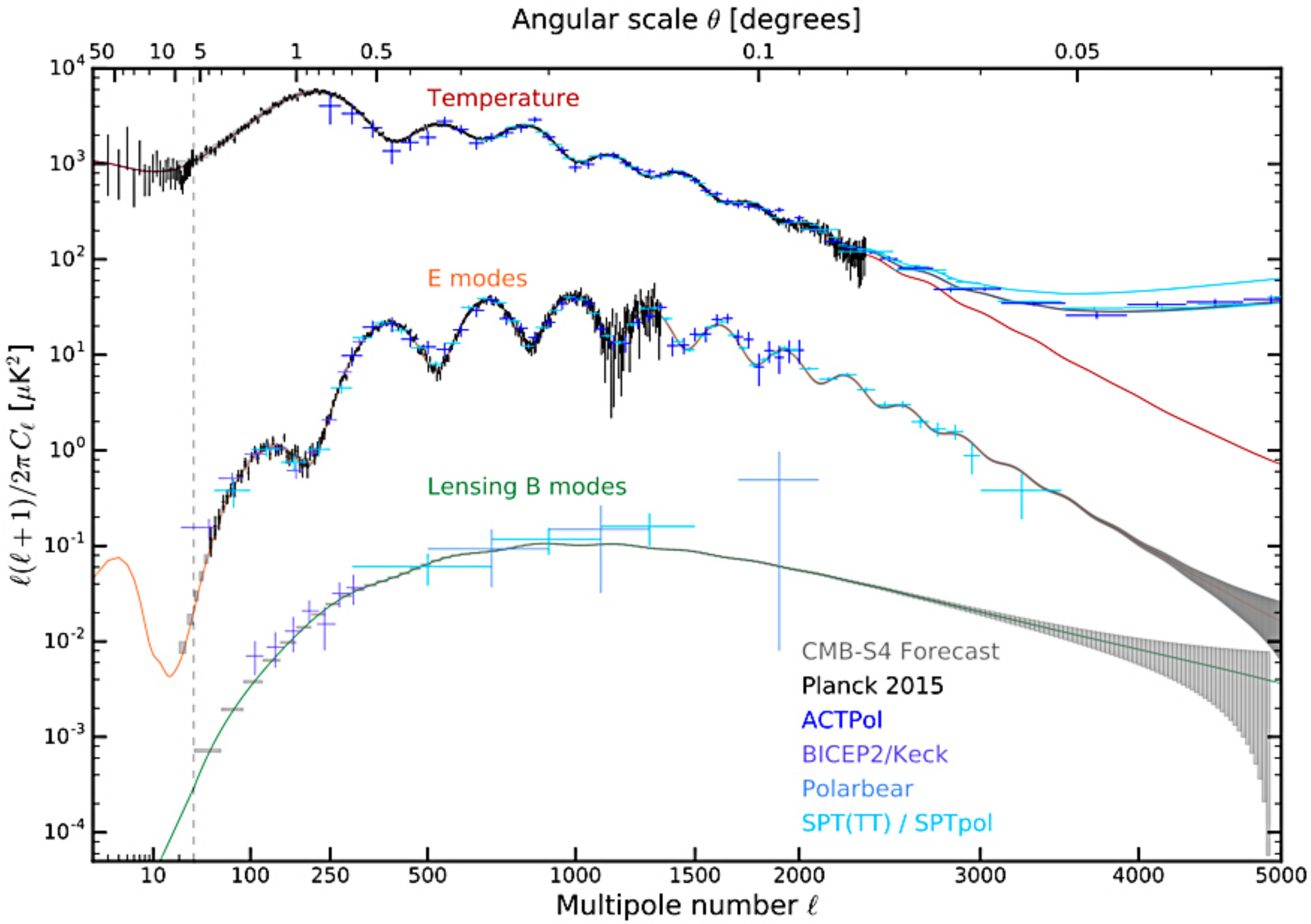
What we want to understand: is it possible to extract this info? How?

Work plan

- **agnostic parametrization** of the astro kernel in terms of few parameters. Forecast of detectability for different observatories: MCMC (noise characterization needed!)
- understand how to relate these parameters to physical astrophysical quantities
- **effective astrophysical model** to capture main dependences: inverse problem for parameters reconstruction

[with I. Michaloliakos, B. Whiting, E. Belllini, D. Alonso]

Can polarization be generated? CMB analogy



Characterization of a GW background: Stokes parameters

Superposition signals in given direction and at a given frequency

strain

$$\tilde{h}_{ij}(f, \mathbf{n}) = \tilde{h}_+(f, \mathbf{n}) e^+_{ij}(\mathbf{n}) + \tilde{h}_\times(f, \mathbf{n}) e^\times_{ij}(\mathbf{n})$$

$$\boxed{\tilde{\mathcal{P}}_{ab} = \tilde{h}^*_a \tilde{h}_b}$$

polarization tensor

$$a, b = + \,, \times$$

It fully describes background

intensity polarization

$$\tilde{\mathcal{P}}_{ab}(\mathbf{n}, f) = \frac{1}{2}\left[I(\mathbf{n}, f)\, 1_{ab} + U(\mathbf{n}, f)\, \sigma^{(1)}_{ab} + V(\mathbf{n}, f)\, \sigma^{(2)}_{ab} + Q(\mathbf{n}, f)\, \sigma^{(3)}_{ab} \right]$$

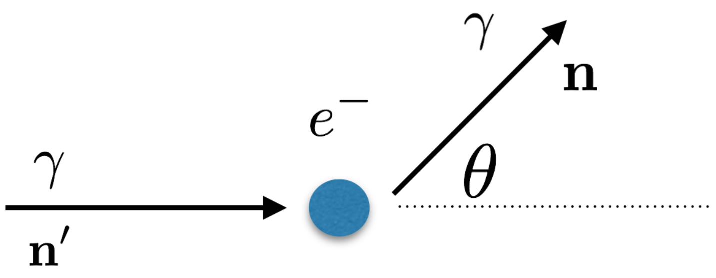
Proportional to background energy density $\Omega_{GW}(\mathbf{n}, f)$

GC, Durrer, Ferreira Phys.Rev. D99 (2019)

Generation of polarization: two ingredients needed

CMB photons

GW background



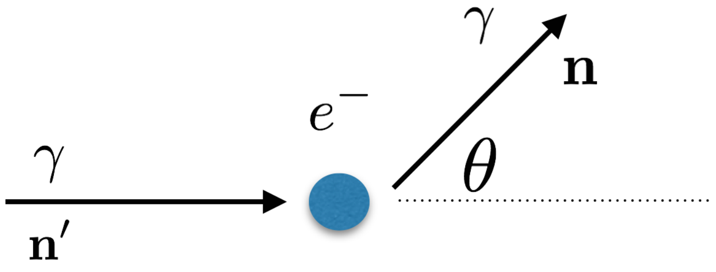
- $$\frac{d\sigma}{d\Omega} = \sigma_T |\epsilon(\mathbf{n})\epsilon(\mathbf{n}')|^2$$
- anisotropy incoming radiation

?

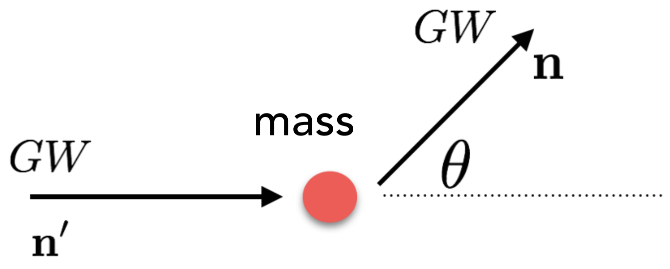
Is there for a GW background a process analogue to Thomson scattering for CMB?

Generation of polarization: wave scattering of gravitons

CMB photons



GW background

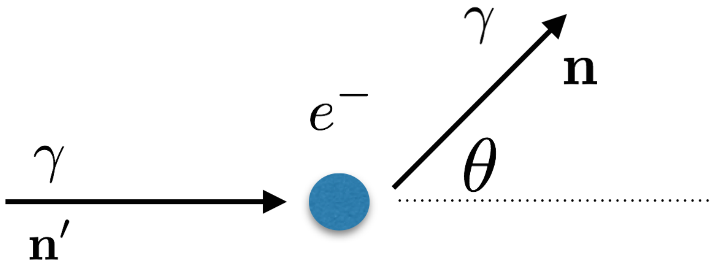


- $$\frac{d\sigma}{d\Omega} = \sigma_T |\epsilon(\mathbf{n})\epsilon(\mathbf{n}')|^2$$
- anisotropy incoming radiation

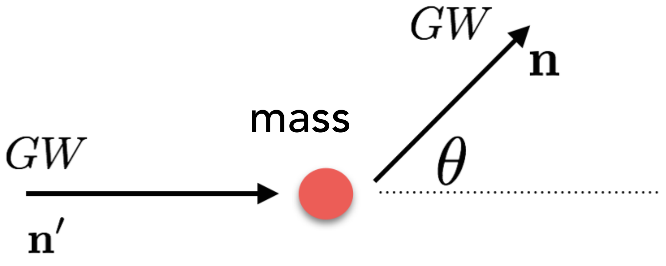
GC, Durrer, Ferreira, Phys.Rev. D99 (2019)

Generation of polarization: wave scattering of gravitons

CMB photons



GW background



- $$\frac{d\sigma}{d\Omega} = \sigma_T |\epsilon(\mathbf{n})\epsilon(\mathbf{n}')|^2$$
- anisotropy incoming radiation

Rutherford-like pre factor

- $$\frac{d\sigma}{d\Omega} = (MG)^2 \frac{1}{\sin^4 \theta/2} |\epsilon_{ij}(\mathbf{n})\epsilon_{ij}(\mathbf{n}')|^2$$
- anisotropy incoming radiation

GC, Durrer, Ferreira, Phys.Rev. D99 (2019)

Diffusion by distribution of lenses

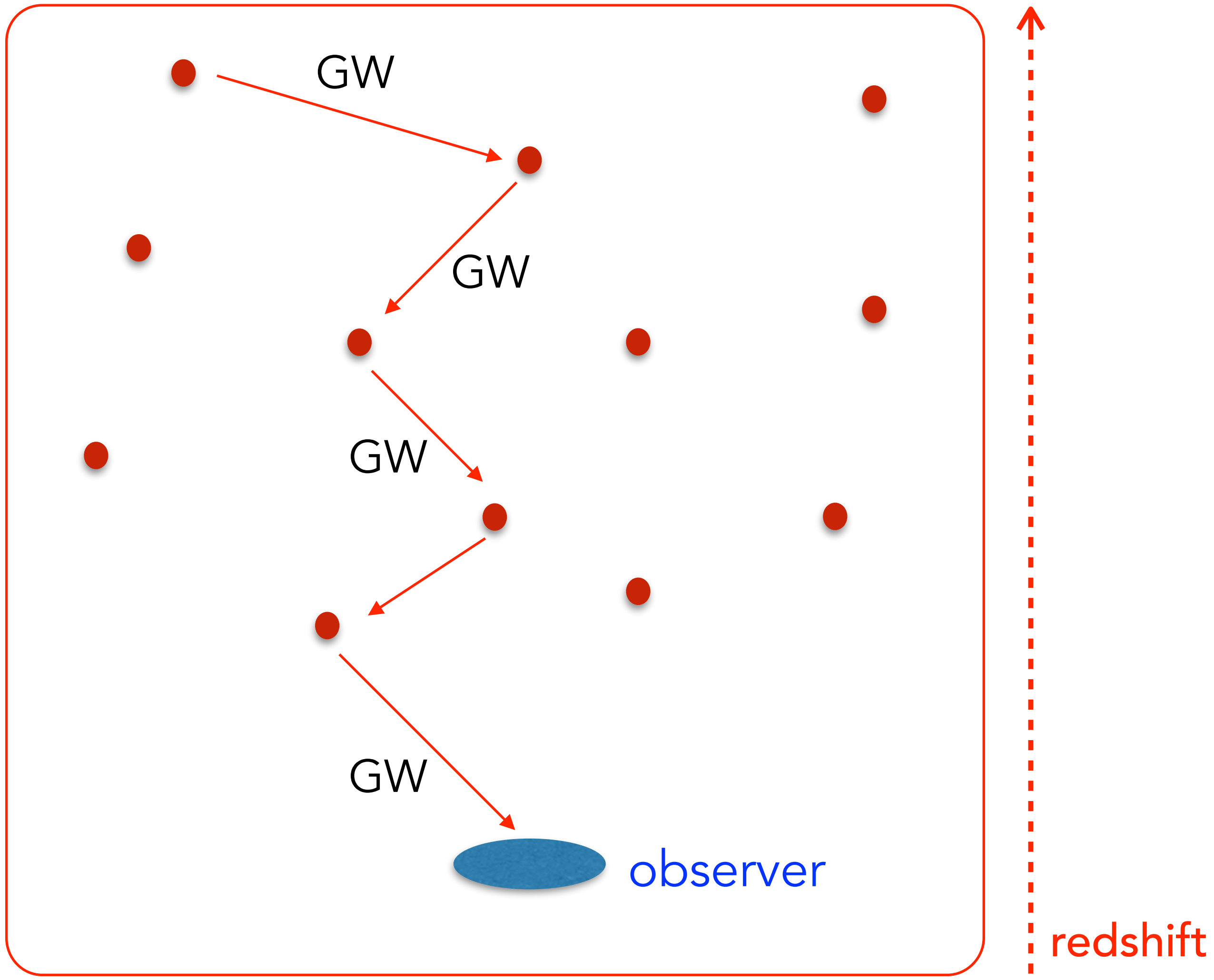
Multi-scattering process with **optical depth**

$$\tau(\eta,\eta_0)=\int_{\eta}^{\eta_0} d\eta' a(\eta')\, n_{\rm ph}(\eta')\sigma(\eta')$$

number density scattering centers

cross section

Visibility function for gravitons extends in redshift (vs CMB)



We derived first **predictions** for the total amount of polarization produced

Typical suppression of a factor $10^{-4}-10^{-3}$ wrt anisotropies [in LISA-PTA bands]

GC, Durrer, Ferreira, Phys.Rev. D99 (2019)

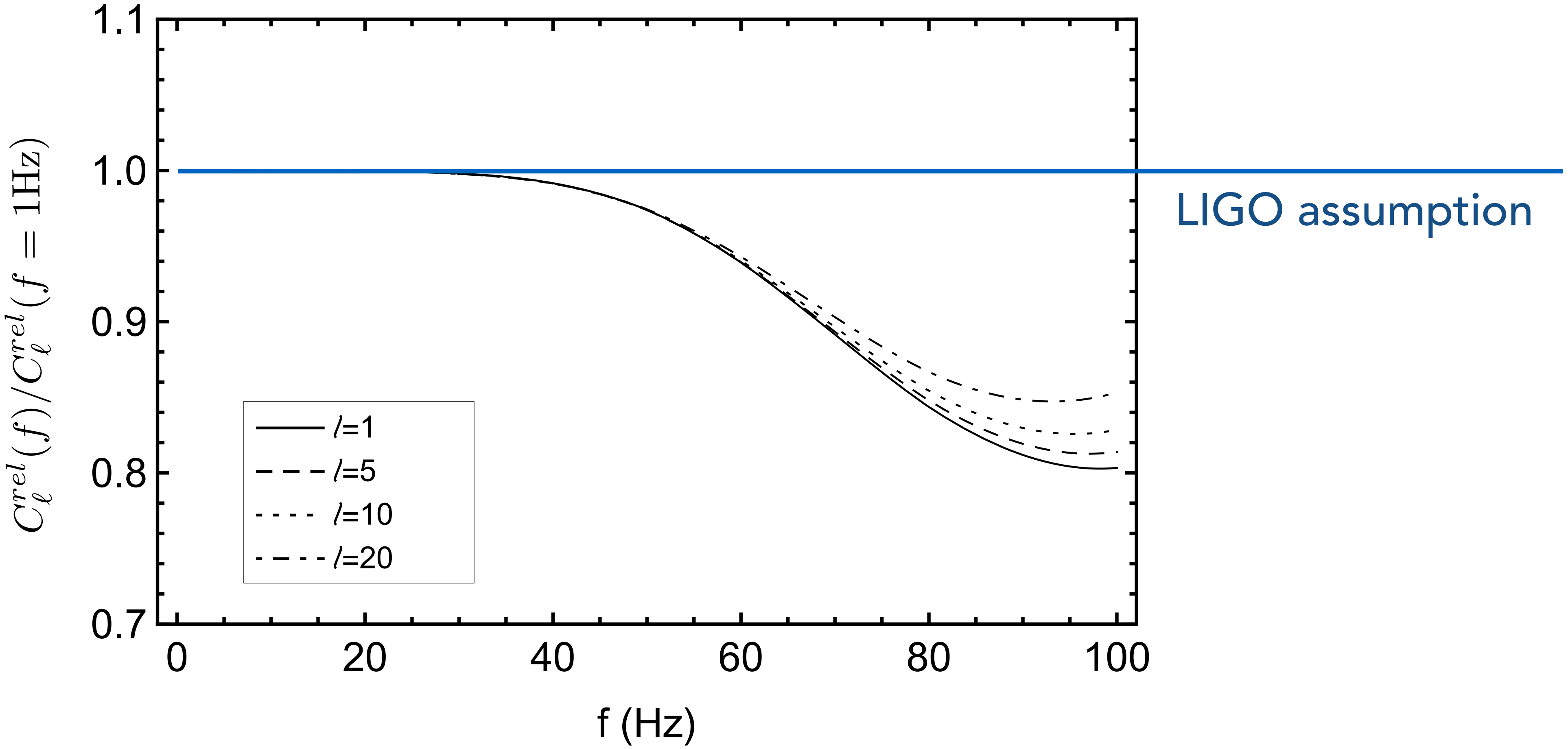
Making contact with observations: characterization of different noise components

Test hypothesis used in current searches

Checking Hp of frequency factorization

$$\Omega_{GW}(\mathbf{e}, f) = \left(\frac{f}{f_{\rm ref}}\right)^{2/3} D(\mathbf{e})$$

angular power spectrum with frequency factor factorized out



LIGO assumption

GC, Dvorkin et al. Phys.Rev. D100 (2019) 063004

Inclusion of noise: (spatial) shot noise

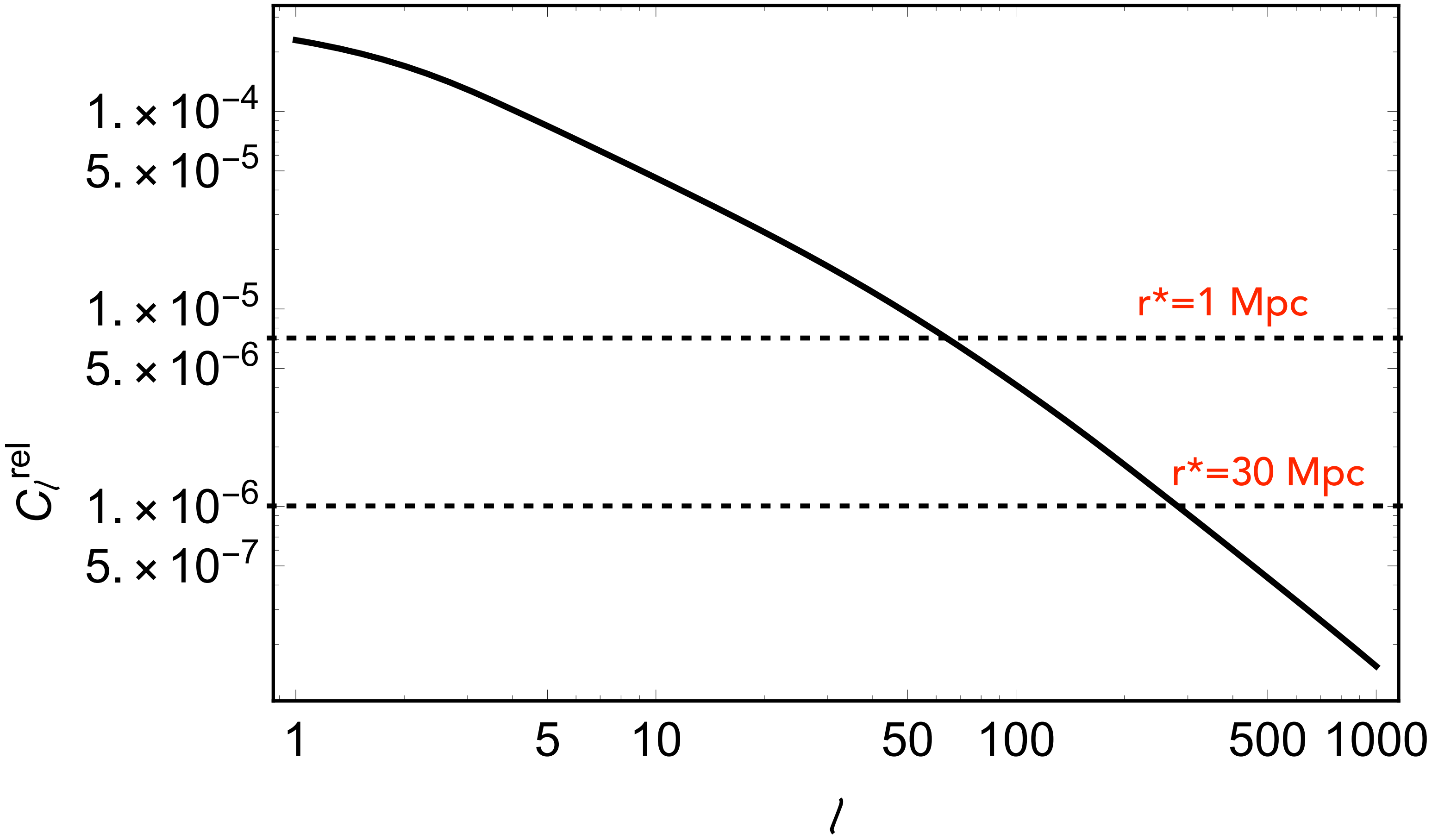
$$\hat{C}_\ell = \boxed{C_\ell} + \boxed{S_n},$$

shot noise

theoretical prediction

$$S_n = \frac{1}{(4\pi)^2}\int_{r^*} dr \left|\frac{\partial\bar{\Omega}_{GW}}{\partial r}\right|^2 \frac{1}{r^2}\,\frac{1}{\bar{n}_{\rm G}(r)}$$

r* corresponds to upper cut-off in flux
(threshold to resolve sources individually)



sources until r* filtered out
(considered resolvable)

GC, Dvorkin et al. Phys.Rev. D100 (2019) 063004

Inclusion of noise: popcorn noise in LIGO band

$$\hat{C}_\ell = \boxed{C_\ell} + \boxed{S_n},$$

popcorn noise

theoretical prediction

$$S_n = \frac{1}{(4\pi)^2} \int_{r^*} dr \left| \frac{\partial \bar{\Omega}_{GW}}{\partial r} \right|^2 \frac{1}{r^2} \frac{1}{\bar{n}_{\mathrm{G}}(r)}$$

$$\downarrow$$

$$\beta_T \cdot \bar{n}_G(r)$$

number of galaxies containing a merger in observation time T

$$\beta_T = \frac{T}{a^3 \bar{n}_{\mathrm{G}}} \times \frac{\mathrm{d}\mathcal{N}}{\mathrm{d}t\mathrm{d}V} \quad <<1$$

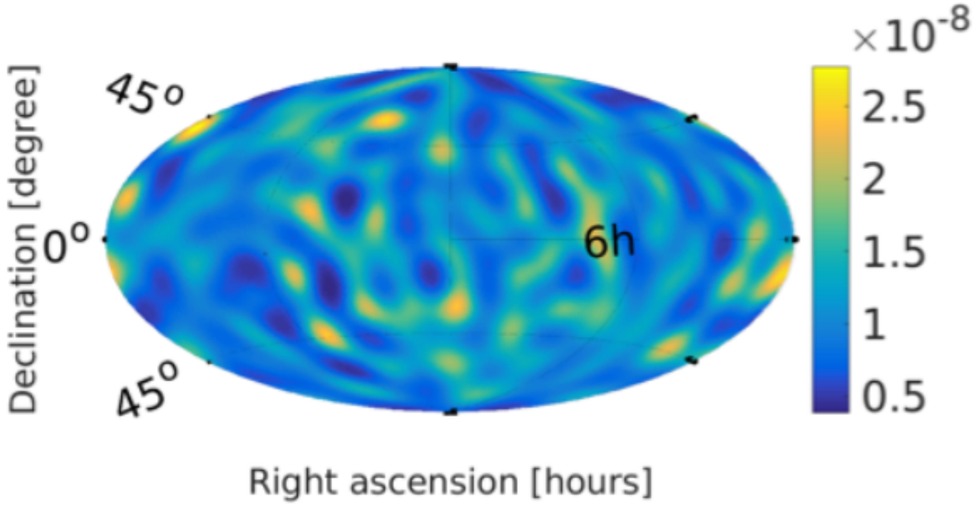
In the frequency band of terrestrial interferometers, there is **large contribution from popcorn noise**

Important! This popcorn noise component **not there in LISA band**. Intrinsic (irreducible) background there

GC, Dvorkin et al. Phys.Rev. D100 (2019) 063004

Cross-correlation (potentially) very useful for popcorn backgrounds

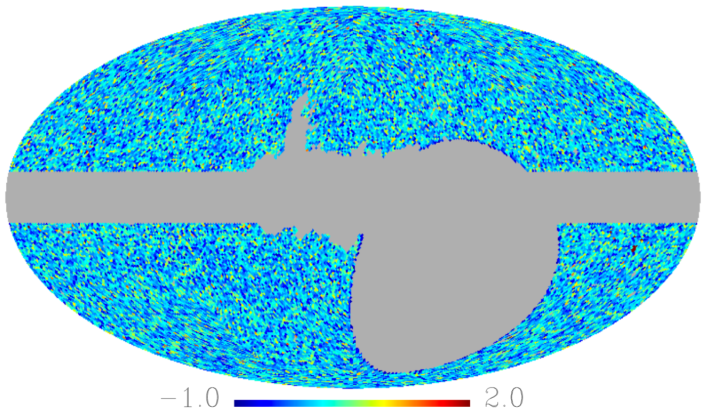
Shot-noise dominated map



(LIGO-Virgo PRL 118, 121102, 2017)

×

Galaxy number counts map



e.g. map radio sources, from: Bengaly, Maartens, Santos 2018

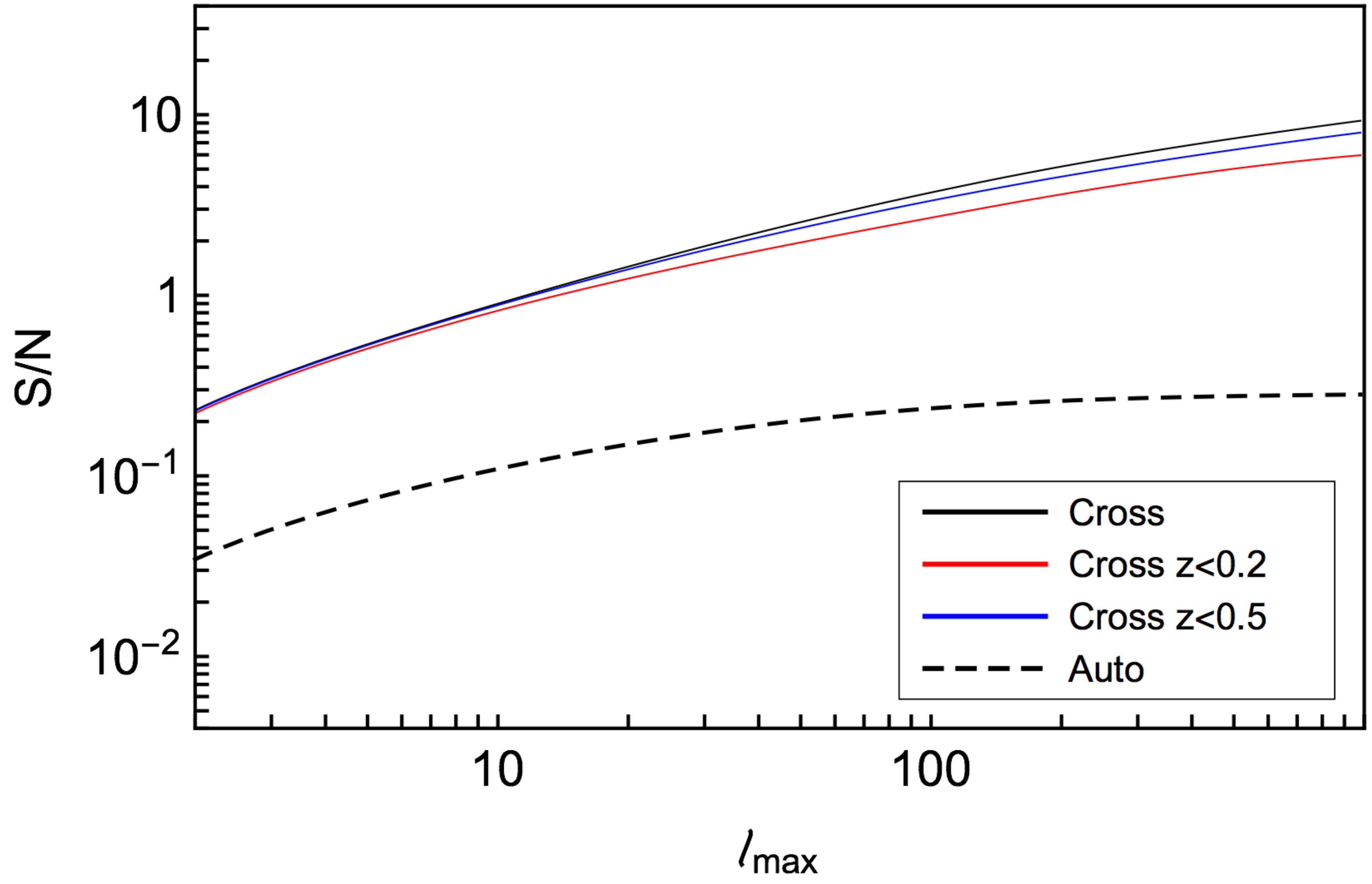
↓

dig GW signal out of popcorn noise threshold

Alonso, **GC,** Ferreira, Pitrou (arXiv 2020)

Power cross-correlation in Hz band: quantitative analysis

Considering popcorn shot noise as only noise contribution



improvement factor >10

Alonso, **GC,** Ferreira, Pitrou (arXiv 2020)

Summary and perspectives

Summary: state of the art and work in progress

Noise characterization, forecasts for present and future missions

	predictions	⟷	data analysis
anisotropies	✓		✓
polarization	✓		Ongoing
cross-correlation (electromagnetic counterparts)	✓		

no existing infrastructure for cross-correlation [ongoing work with Mandic, Scarlata]

first theoretical predictions

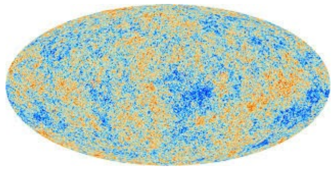
we know: there is astro info inside anisotropies

how to extract it?

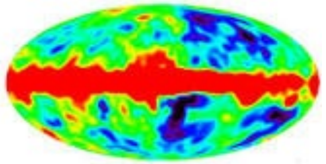
Cosmic Microwave Background (CMB)

angular resolution

observations



WMAP '03
Planck '13

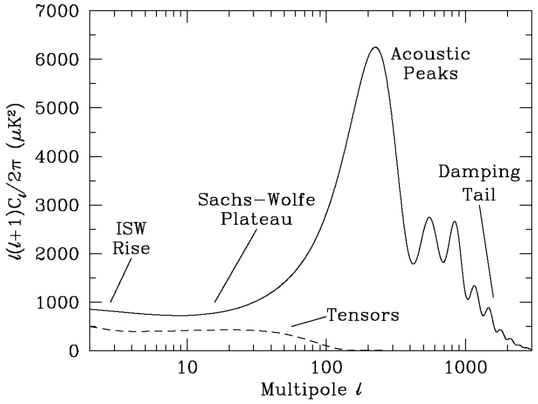


COBE '89

Penzias, Wilson '65

Precision cosmology

theoretical predictions
Sachs and Wolfe '67
Rees and Sciama '68
…

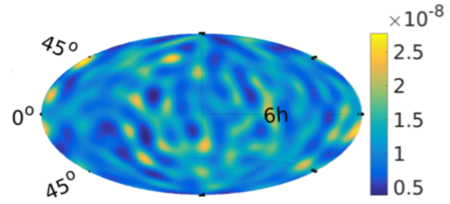


Astrophysical GW background

angular resolution

no detection yet! upper bounds

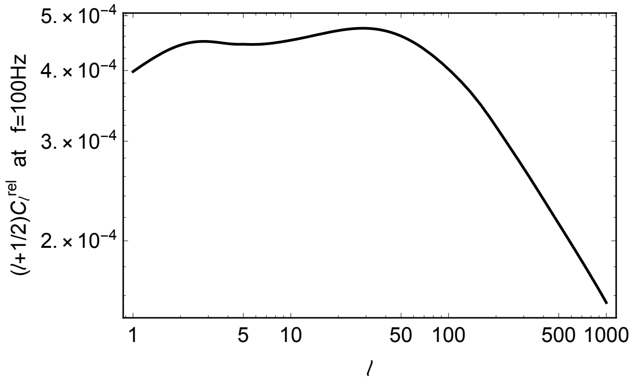
LIGO-Virgo, PTA



LISA, ET

Precision astrophysics (?)

we have the first theoretical predictions here!

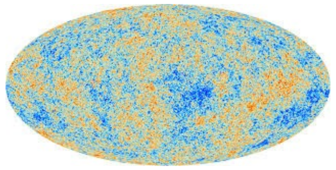


Cosmic Microwave Background (CMB)

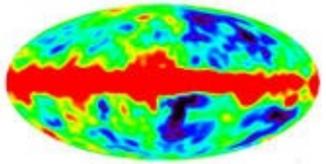
Astrophysical GW background

angular resolution ⟷ angular resolution

observations



WMAP ’03
Planck ‘13



COBE ’89

Penzias, Wilson ‘65

↓

Precision cosmology

↑

theoretical predictions
Sachs and Wolfe ’67
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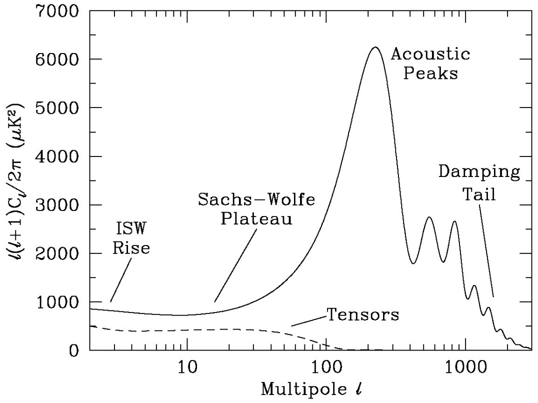
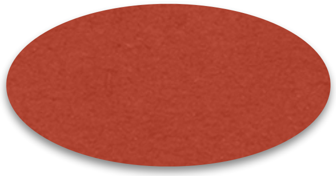
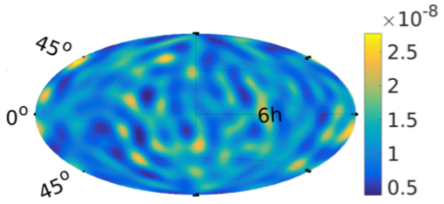


no detection yet! upper bounds



LIGO-Virgo, PTA



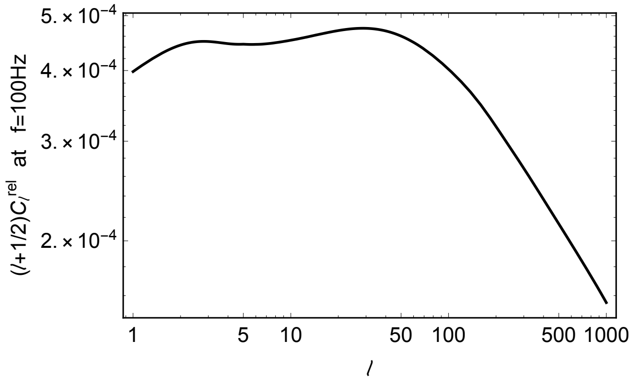
LISA-ET

↓

Precision astrophysics (?)

↑

we have the first theoretical predictions here!



This study is also necessary for foreground subtraction —-> extract cosmological background of GW

Thank you