Two-body problem in modified gravities and effective-one-body theory

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Motivations

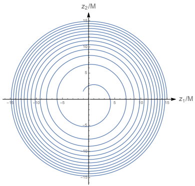
• GW150914 : The very first observation of a BBH coalescence by LIGO-Virgo has opened **a new era in gravitational wave astronomy.**

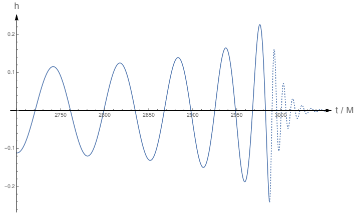
• Opportunity to bring **new tests of modified gravities**, in the strong-field regime near merger.

• EOB is a powerful approach to describe **analytically** the coalescence of 2 compact objects in **General Relativity**, from inspiral **to merger**.

$$H(Q,P)\ , \quad \epsilon = \left(\frac{v}{c}\right)^2 \qquad \longrightarrow \qquad H_e(q,p)\ , \quad ds_e^2 = g^e_{\mu\nu} dx^\mu dx^\nu$$

$$\boxed{H_e = f_{\mathrm{EOB}}(H)}$$





• Instrumental to build libraries of waveform templates for LIGO-VIRGO

FLJ, Nathalie Deruelle [Phys.Rev. D95, 124054]

- Can we extend the EOB approach to modified gravities ?
- A simple example: **massless scalar-tensor theories**.

- First buiding block : map the conservative part of the two-body dynamics onto the geodesic of an effective metric.

- ST-extension of [Buonanno-Damour 98]

ST action in the Einstein-frame ($G_* \equiv c \equiv 1$)

$$S_{EF} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \Big(R - 2 g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \Big) + S_m \left[\Psi, \mathcal{A}^2(\varphi) g_{\mu\nu} \right]$$

Skeletonization of compact bodies :

$$S_m = -\sum_A \int d\lambda \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}} \, m_A(\varphi)$$

$m_A(\varphi)$ depends on the theory $\mathcal{A}(\varphi)$ and on the EOS of body A.

$\rightarrow$ Strong Equivalence Principle violation [Eardley 75, DEF 92]

Our starting point : what is known today

Two-body Scalar-Tensor Lagrangian [DEF 93][Mirshekari, Will 13]

- conservative 2PK dynamics : $\mathcal{O}\left(\left(\frac{v}{c}\right)^4\right) \sim \mathcal{O}\left(\left(\frac{m}{r}\right)^2\right)$ corrections to Kepler
- Weak field expansion

$$
\begin{aligned}
g_{\mu\nu} &= \eta_{\mu\nu} + \delta g_{\mu\nu} \\
\varphi &= \varphi_0 + \delta\varphi
\end{aligned}
$$

- the fundamental functions $m_A(\varphi)$ and $m_B(\varphi)$ are expanded around φ_0 :

$$
\begin{aligned}
\ln m_A(\varphi) &\equiv \ln m_A^0 + \alpha_A^0(\varphi - \varphi_0) + \beta_A^0(\varphi - \varphi_0)^2 + {\beta'}_A^0(\varphi - \varphi_0)^3 + \cdots \\
\ln m_B(\varphi) &\equiv \ln m_B^0 + \alpha_B^0(\varphi - \varphi_0) + \beta_B^0(\varphi - \varphi_0)^2 + {\beta'}_B^0(\varphi - \varphi_0)^3 + \cdots
\end{aligned}
$$

i.e. the 2PK Lagrangian depends on 8 fundamental **parameters**.

Two-body 2PK Lagrangian

$$L = -m_A^0 - m_B^0 + L_{\rm K} + L_{\rm 1PK} + L_{\rm 2PK} + \cdots$$

$$\vec{N} \equiv \frac{\vec{Z}_A - \vec{Z}_B}{R}\,, \quad \vec{V}_A \equiv \frac{d\vec{Z}_A}{dt}\,, \quad R \equiv \mid \vec{Z}_A - \vec{Z}_B \mid\,, \quad \vec{A}_A \equiv \frac{d\vec{V}_A}{dt}$$

- Keplerian order :

$$\boxed{L_{\rm K} = \frac{1}{2} m_A^0 V_A^2 + \frac{1}{2} m_B^0 V_B^2 + \frac{G_{AB} m_A^0 m_B^0}{R}} \qquad \text{where} \qquad G_{AB} \equiv 1 + \alpha_A^0 \alpha_B^0$$

- post-Keplerian (1PK) :

$$\begin{aligned} L_{\rm 1PK} &= \frac{1}{8} m_A^0 V_A^4 + \frac{1}{8} m_B^0 V_B^4 \\ &+ \frac{G_{AB} m_A^0 m_B^0}{R} \left(\frac{3}{2}(V_A^2 + V_B^2) - \frac{7}{2} \vec{V}_A \cdot \vec{V}_B - \frac{1}{2} (\vec{N} \cdot \vec{V}_A)(\vec{N} \cdot \vec{V}_B) + \bar{\gamma}_{AB} (\vec{V}_A - \vec{V}_B)^2 \right) \\ &- \frac{G_{AB}^2 m_A^0 m_B^0}{2R^2} \left(m_A^0 (1 + 2\bar{\beta}_B) + m_B^0 (1 + 2\bar{\beta}_A) \right) \end{aligned}$$

where $\bar{\gamma}_{AB} \equiv -\frac{2\alpha_A^0 \alpha_B^0}{1+\alpha_A^0 \alpha_B^0} \quad \bar{\beta}_A \equiv \frac{1}{2} \frac{\beta_A^0 (\alpha_B^0)^2}{(1+\alpha_A^0 \alpha_B^0)^2} \; (A \leftrightarrow B)$

- post-post-Keplerian (2PK) :

$$
\begin{aligned}
L_{\rm 2PK} &= \frac{1}{16} m_A^0 V_A^6 \\
&+ \frac{G_{AB} m_A^0 m_B^0}{R} \left[\frac{1}{8}(7+4\bar\gamma_{AB}) \left(V_A^4 - V_A^2 (\vec N \cdot \vec V_B)^2 \right) - (2+\bar\gamma_{AB}) V_A^2 (\vec V_A \cdot \vec V_B) + \frac{1}{8} (\vec V_A \cdot \vec V_B)^2 \right. \\
&\quad \left. + \frac{1}{16}(15+8\bar\gamma_{AB}) V_A^2 V_B^2 + \frac{3}{16} (\vec N \cdot \vec V_A)^2 (\vec N \cdot \vec V_B)^2 + \frac{1}{4}(3+2\bar\gamma_{AB}) \vec V_A \cdot \vec V_B (\vec N \cdot \vec V_A)(\vec N \cdot \vec V_B) \right] \\
&+ \frac{G_{AB}^2 m_B^0 (m_A^0)^2}{R^2} \left[\frac{1}{8} \left(2 + 12\bar\gamma_{AB} + 7\bar\gamma_{AB}^2 + 8\bar\beta_B - 4\delta_A \right) V_A^2 + \frac{1}{8} \left(14 + 20\bar\gamma_{AB} + 7\bar\gamma_{AB}^2 + 4\bar\beta_B - 4\delta_A \right) V_B^2 \right. \\
&\quad - \frac{1}{4} \left(7 + 16\bar\gamma_{AB} + 7\bar\gamma_{AB}^2 + 4\bar\beta_B - 4\delta_A \right) \vec V_A \cdot \vec V_B - \frac{1}{4} \left(14 + 12\bar\gamma_{AB} + \bar\gamma_{AB}^2 - 8\bar\beta_B + 4\delta_A \right) (\vec V_A \cdot \vec N)(\vec V_B \cdot \vec N) \\
&\quad \left. + \frac{1}{8} \left(28 + 20\bar\gamma_{AB} + \bar\gamma_{AB}^2 - 8\bar\beta_B + 4\delta_A \right) (\vec N \cdot \vec V_A)^2 + \frac{1}{8} \left(4 + 4\bar\gamma_{AB} + \bar\gamma_{AB}^2 + 4\delta_A \right) (\vec N \cdot \vec V_B)^2 \right] \\
&+ \frac{G_{AB}^3 (m_A^0)^3 m_B^0}{2R^3} \left[1 + \frac{2}{3}\bar\gamma_{AB} + \frac{1}{6}\bar\gamma_{AB}^2 + 2\bar\beta_B + \frac{2}{3}\delta_A + \frac{1}{3}\epsilon_B \right] + \frac{G_{AB}^3 (m_A^0)^2 (m_B^0)^2}{8R^3} \left[19 + 8\bar\gamma_{AB} + 8(\bar\beta_A + \bar\beta_B) + 4\zeta \right] \\
&- \frac{1}{8} G_{AB} m_A^0 m_B^0 \left(2(7+4\bar\gamma_{AB}) \vec A_A \cdot \vec V_B (\vec N \cdot \vec V_B) + \vec N \cdot \vec A_A (\vec N \cdot \vec V_B)^2 - (7+4\bar\gamma_{AB}) \vec N \cdot \vec A_A V_B^2 \right) \\
&\quad + (A \leftrightarrow B)
\end{aligned}
$$

where $\delta_A \equiv \frac{(\alpha_A^0)^2}{(1+\alpha_A^0\alpha_B^0)^2}$ $\quad \epsilon_A \equiv \frac{(\beta'_A \alpha_B^3)^0}{(1+\alpha_A^0\alpha_B^0)^3}$ $\quad \zeta \equiv \frac{\beta_A^0 \alpha_A^0 \alpha_B^0 \beta_B^0}{(1+\alpha_A^0\alpha_B^0)^3}$ $(A \leftrightarrow B)$

Geodesic motion in a static, spherically symmetric metric

In Schwarzschild-Droste coordinates (equatorial plane $\theta = \pi/2$) :

$$\boxed{ds_e^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\phi^2}$$

$A(r)$ and $B(r)$ are arbitrary.

Effective Hamiltonian $H_e(q,p)$:

$$H_e(q,p) = \sqrt{A\left(\mu^2 + \frac{p_r^2}{B} + \frac{p_\phi^2}{\hat{r}^2}\right)} \quad \text{with} \quad p_r \equiv \frac{\partial L_e}{\partial \dot{r}} \quad , \quad p_\phi \equiv \frac{\partial L_e}{\partial \dot{\phi}}$$

Can be **expanded** :

$$\boxed{\begin{aligned} A(r) &= 1 + \frac{a_1}{r} + \frac{a_2}{r^2} + \frac{a_3}{r^3} + \cdots \\ B(r) &= 1 + \frac{b_1}{r} + \frac{b_2}{r^2} + \cdots \end{aligned}}$$

i.e. depends on **5 effective parameters** at 2PK order, to be determined.

1) Use of a canonical transformation :

$$H(Q,P) \to H(q,p)$$

Generic ansatz $G(Q,p)$ that depends on 9 parameters at 2PK order :

$$G(Q,p) = R\,p_r \left[\left(\alpha_1 \mathcal{P}^2 + \beta_1 \hat{p}_r^2 + \frac{\gamma_1}{\hat{R}} \right) + \left(\alpha_2 \mathcal{P}^4 + \beta_2 \mathcal{P}^2 \hat{p}_r^2 + \gamma_2 \hat{p}_r^4 + \delta_2 \frac{\mathcal{P}^2}{\hat{R}} + \epsilon_2 \frac{\hat{p}_r^2}{\hat{R}} + \frac{\eta_2}{\hat{R}^2} \right) + \cdots \right]$$

2) Relate H to H_e through the quadratic relation [Damour 2016]

$$\frac{H_e(q,p)}{\mu} - 1 = \left(\frac{H(q,p) - M}{\mu} \right) \left[1 + \frac{\nu}{2} \left(\frac{H(q,p) - M}{\mu} \right) \right]$$

where $\nu = \frac{m_A^0 m_B^0}{(m_A^0 + m_B^0)^2}$, $M = m_A^0 + m_B^0$, $\mu = \frac{m_A^0 m_B^0}{M}$

The Scalar-Tensor effective metric

$$ds_e^2 = -A(r)dt + B(r)dr^2 + r^2 d\theta^2$$

Yields a **unique** solution in scalar-tensor theories (coordinate-independent)

Scalar-Tensor effective metric

$$A(r) = 1 - 2\left(\frac{G_{AB}M}{r}\right) + 2\left[\langle\bar{\beta}\rangle - \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \left[2\nu + \delta a_3^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^3 + \cdots$$

$$B(r) = 1 + 2\left[1 + \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right) + \left[2(2-3\nu) + \delta b_2^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \cdots$$

Reduces to GR when $m_A(\varphi) = cst$

General Relativity 2PN effective metric [Buonanno, Damour 98]

$$A_{\mathrm{GR}}(r) = 1 - 2\left(\frac{G_* M}{r}\right) + 2\nu\left(\frac{G_* M}{r}\right)^3 + \cdots$$

$$B_{\mathrm{GR}}(r) = 1 + 2\left(\frac{G_* M}{r}\right) + 2(2-3\nu)\left(\frac{G_* M}{r}\right)^2 + \cdots$$

Scalar-Tensor effective metric

$$A(r) = 1 - 2\left(\frac{G_{AB}M}{r}\right) + 2\left[\langle\bar{\beta}\rangle - \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \left[2\nu + \delta a_3^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^3 + \cdots$$

$$B(r) = 1 + 2\left[1 + \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right) + \left[2(2-3\nu) + \delta b_2^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \cdots$$

(iii) 2PK corrections

$$\delta a_3^{\mathrm{ST}} \equiv \frac{1}{12}\Big[-20\bar{\gamma}_{AB} - 35\bar{\gamma}_{AB}^2 - 24\langle\bar{\beta}\rangle(1 - 2\bar{\gamma}_{AB}) + 4\big(\langle\delta\rangle - \langle\epsilon\rangle\big)$$
$$+\nu\Big(-36(\bar{\beta}_A + \bar{\beta}_B) + 4\bar{\gamma}_{AB}(10 + \bar{\gamma}_{AB}) + 4(\epsilon_A + \epsilon_B) + 8(\delta_A + \delta_B) - 24\zeta\Big)\Big]$$

$$\delta b_2^{\mathrm{ST}} \equiv \Big[4\langle\bar{\beta}\rangle - \langle\delta\rangle + \bar{\gamma}_{AB}(9 + \frac{19}{4}\bar{\gamma}_{AB}) + \nu\Big(2\langle\bar{\beta}\rangle - 4\bar{\gamma}_{AB}\Big)\Big]$$

$$\delta_A \equiv \frac{(\alpha_A^0)^2}{(1+\alpha_A^0\alpha_B^0)^2} \qquad \epsilon_A \equiv \frac{(\beta'_A\alpha_B^3)^0}{(1+\alpha_A^0\alpha_B^0)^3} \qquad \zeta \equiv \frac{\beta_A^0\alpha_A^0\alpha_B^0\beta_B^0}{(1+\alpha_A^0\alpha_B^0)^3}$$

• The inversion of $H_e = f_{\rm EOB}(H)$ defines a "resummed" EOB Hamiltonian :

$$H_{\rm EOB} = M\sqrt{1+2\nu\left(\frac{H_e}{\mu}-1\right)} \quad \text{where} \quad H_e = \sqrt{A\left(\mu^2+\frac{p_r^2}{B}+\frac{p_\phi^2}{r^2}\right)}$$

• $H_{\rm EOB}$ defines a resummed dynamics, that may capture some features of the strong field regime.

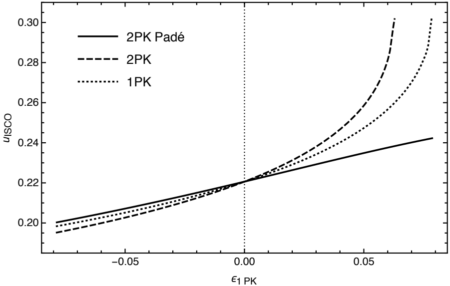
A typical strong-field feature : ST corrections to orbital frequency at the ISCO (equal-mass case : $\nu = 1/4$)

A typical strong-field feature : orbital frequency at the ISCO, equal-mass case ($\nu = 1/4$), setting $\epsilon_{1\mathrm{PK}} \equiv \epsilon^0_{2\mathrm{PK}} \equiv \epsilon^\nu_{2\mathrm{PK}}$

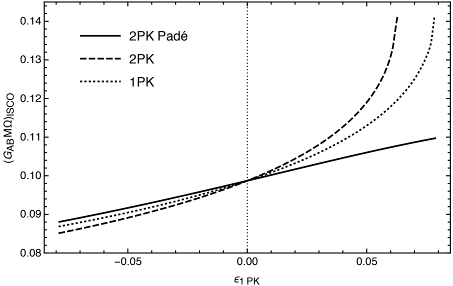
- 2PK Padeed corrections,

$$A = \mathcal{P}^1_5[A^{\mathrm{GR}}_{\mathrm{EOBNR}}(u\,;\nu) + 2\epsilon_{1\mathrm{PK}}u^2 + (\epsilon^0_{2\mathrm{PK}} + \nu\epsilon^\nu_{2\mathrm{PK}})u^3]$$

$u_{\mathrm{ISCO}}(\epsilon_{1\mathrm{PK}})$



$G_{AB}M\Omega_{\mathrm{ISCO}}(\epsilon_{1\mathrm{PK}})$



$$\left.\frac{d(G_{AB}M\Omega)_{ISCO}}{d\epsilon_{1\mathrm{PK}}}\right|_{\nu=1/4} \simeq 0.13$$

relative correction to GR significant ($\sim 10\%$) when $\epsilon_{1\mathrm{PK}} \sim 10^{-2} - 10^{-1}$

Concluding remarks :

• Remarkably, the EOB approach is valid beyond the scope of General Relativity. In **Scalar-Tensor theories :**

$$A^{\mathrm{2PK}}(u) \equiv \mathcal{P}_5^1[A_{\mathrm{5PN}}^{\mathit{Taylor}} + 2\epsilon_{\mathrm{1PK}} u^2 + (\epsilon_{\mathrm{2PK}}^0 + \nu\, \epsilon_{\mathrm{2PK}}^{\nu}) u^3]$$

• The Scalar-Tensor example suggests a generic 2PK ansatz

$$\boxed{A^{\mathrm{PEOB}}(u) \equiv \mathcal{P}_5^1[A_{\mathrm{5PN}}^{\mathit{Taylor}} + 2(\epsilon_{\mathrm{1PK}}^0 + \nu\, \epsilon_{\mathrm{1PK}}^{\nu}) u^2 + (\epsilon_{\mathrm{2PK}}^0 + \nu\, \epsilon_{\mathrm{2PK}}^{\nu}) u^3]}$$

where $\epsilon_{\mathrm{1PK}}^0$, $\epsilon_{\mathrm{1PK}}^{\nu}$, $\epsilon_{\mathrm{2PK}}^0$, and $\epsilon_{\mathrm{2PK}}^{\nu}$ are theory-agnostic Parametrized EOB (PEOB) coefficients.

• Recent work: Einstein-Maxwell-dilaton theories

EMD action in the Einstein-frame ($G_* \equiv c \equiv 1$)

$$S_{\mathrm{EMD}} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(R - 2g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - e^{-2a\varphi} F^{\mu\nu} F_{\mu\nu} \right) + S_m \left[\Psi, \mathcal{A}^2(\varphi) g_{\mu\nu}, A_\mu \right]$$