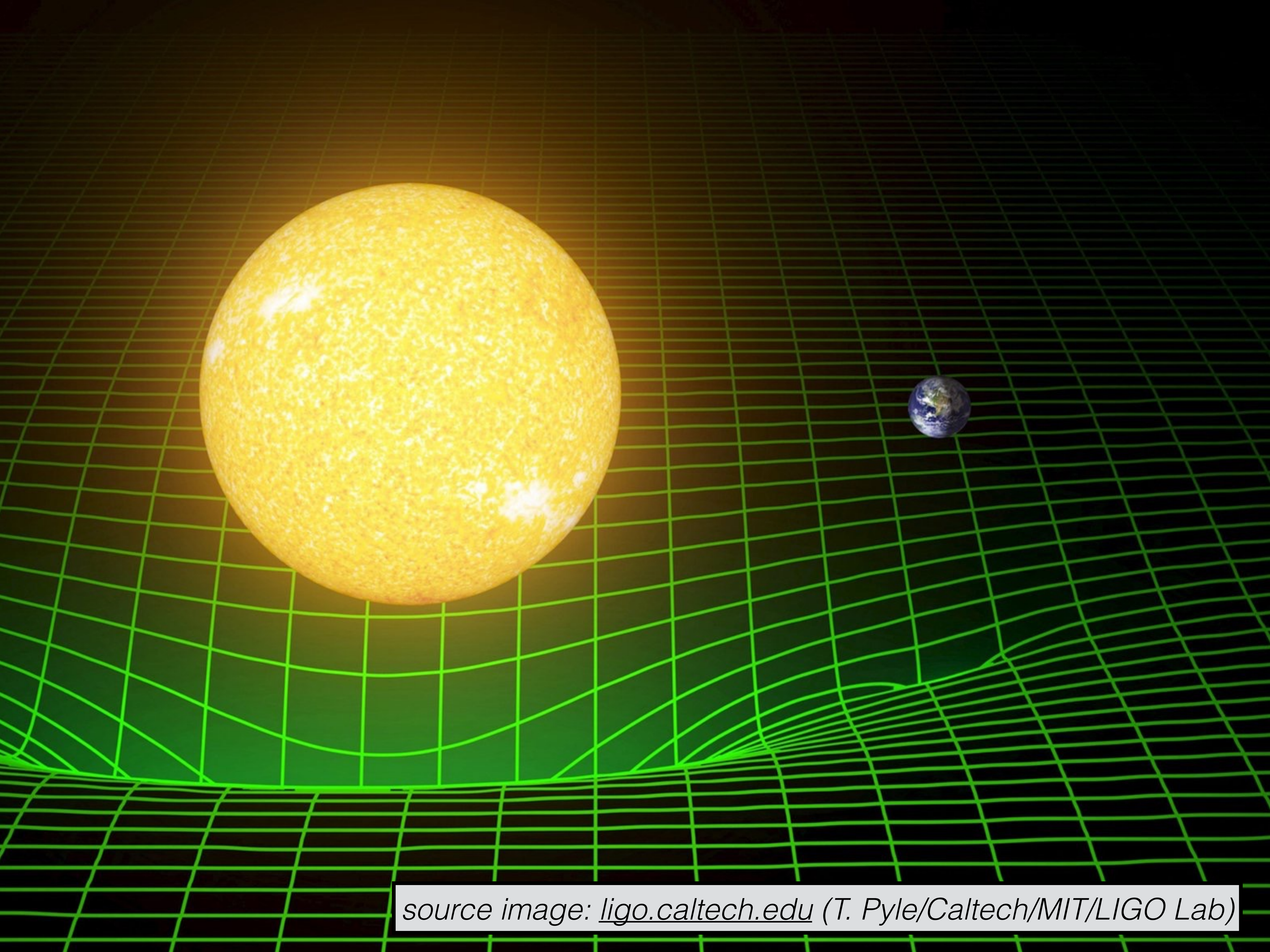
Post-Newtonian theory and gravitational waves

Tanguy Marchand

David Langlois (APC) - Luc Blanchet (IAP)

Journée des thèses - APC - 10 Novembre 2016



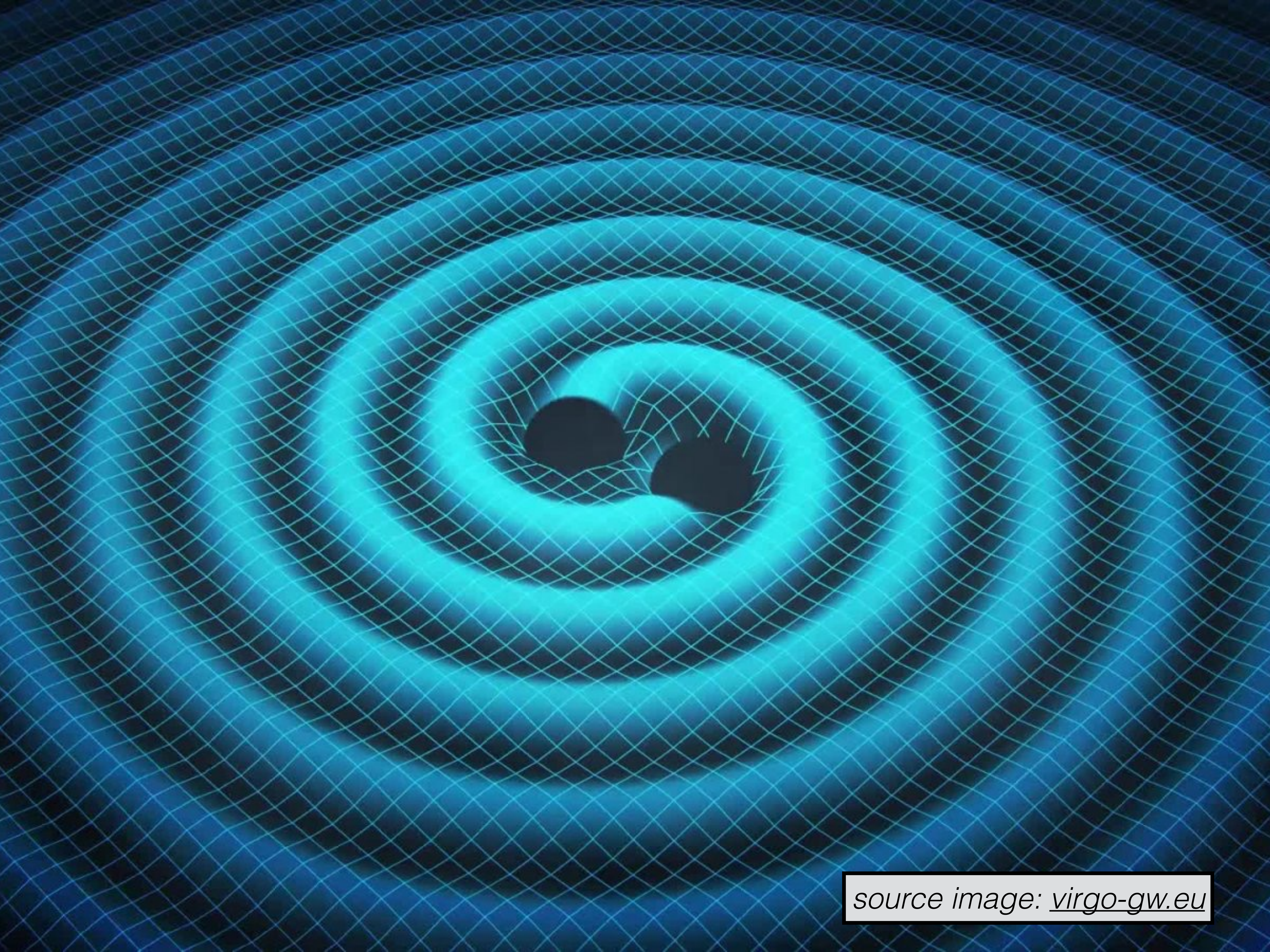
source image: ligo.caltech.edu (T. Pyle/Caltech/MIT/LIGO Lab)

General relativity in a nutshell

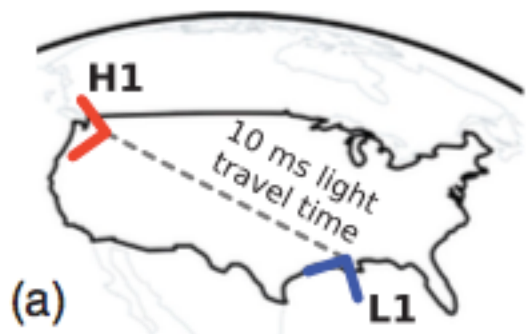
- The spacetime is a manifold of dimension 4 with a metric g_{ab} (which is a symmetric rank 2 tensor)

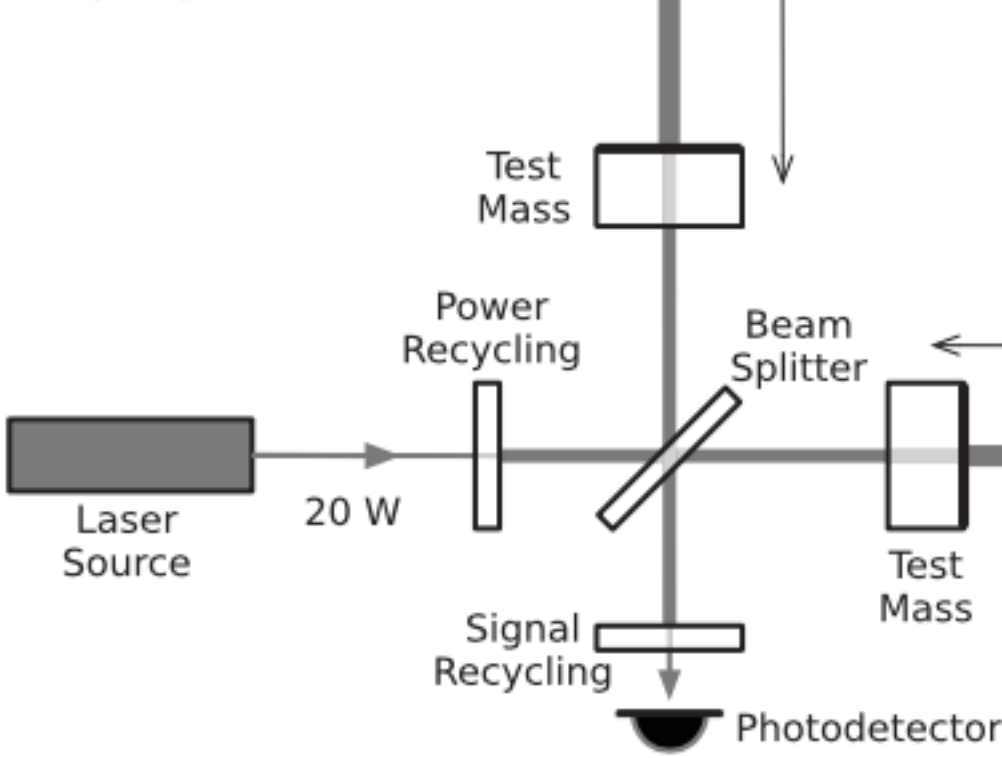
- $$G_{\mu\nu}(g_{\alpha\beta}, \partial g_{\alpha\beta}, \partial^2 g_{\alpha\beta}) = \frac{8\pi G}{c^4} T_{\mu\nu}$$

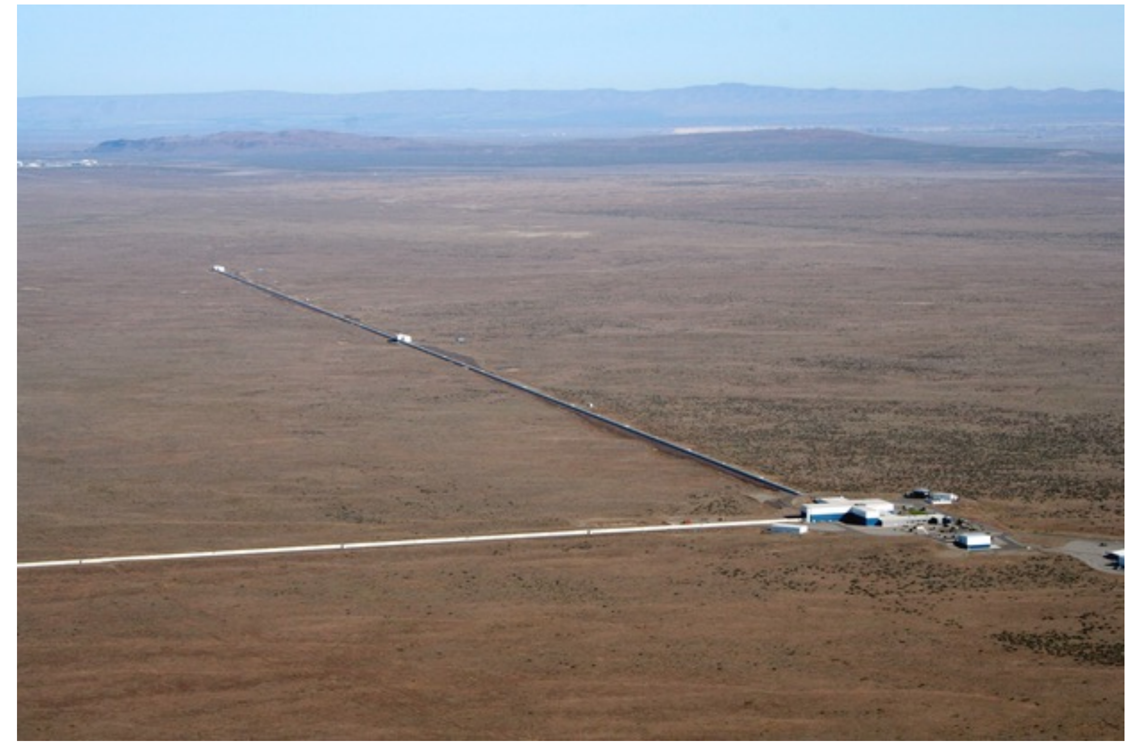
- Matter follows **geodesics** in this curved spacetime


source image: virgo-gw.eu

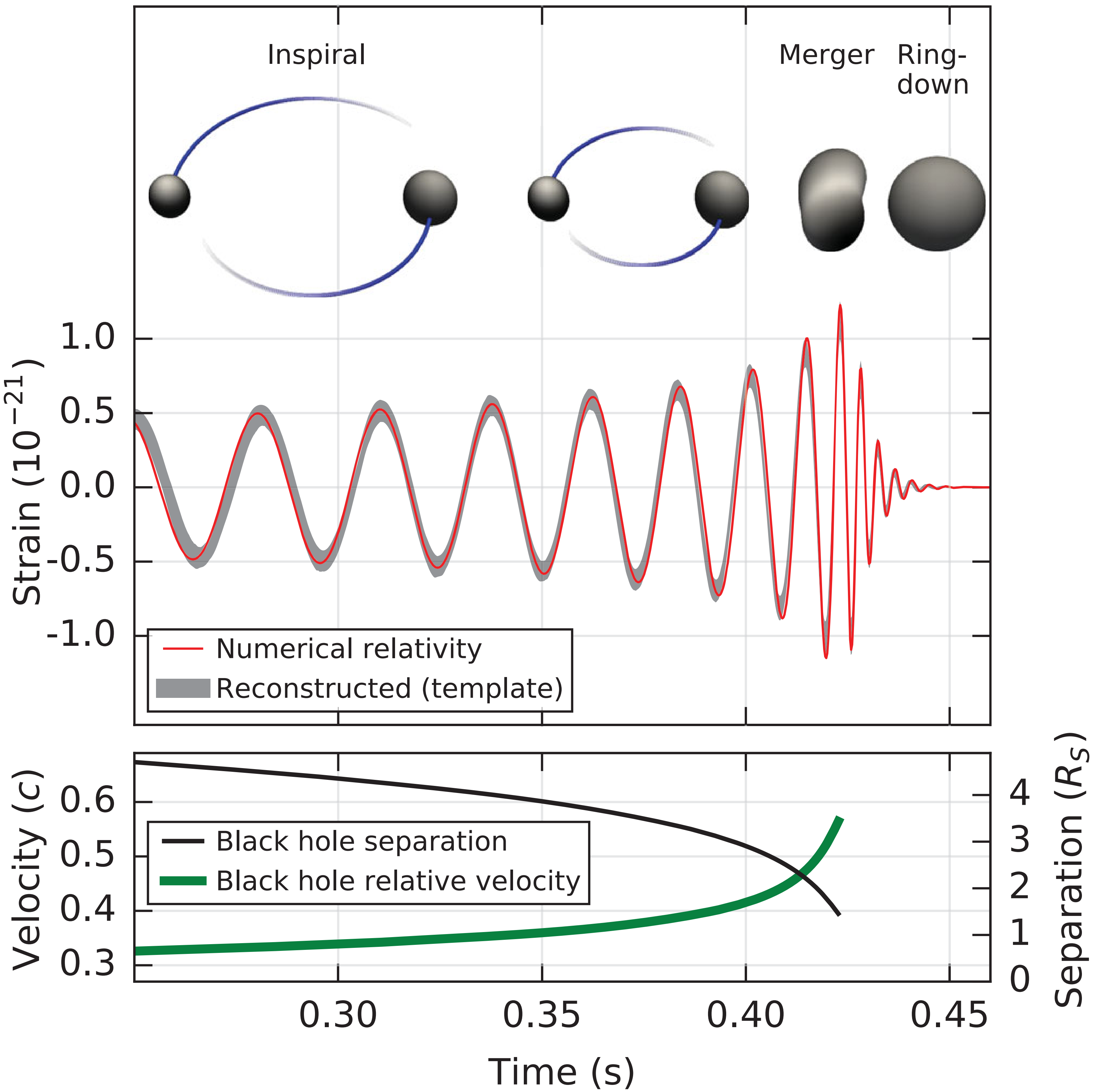
Credit: P. S. Shawhan for the LIGO Scientific Collaboration and Virgo Collaboration (cf arxiv:1210.7173)



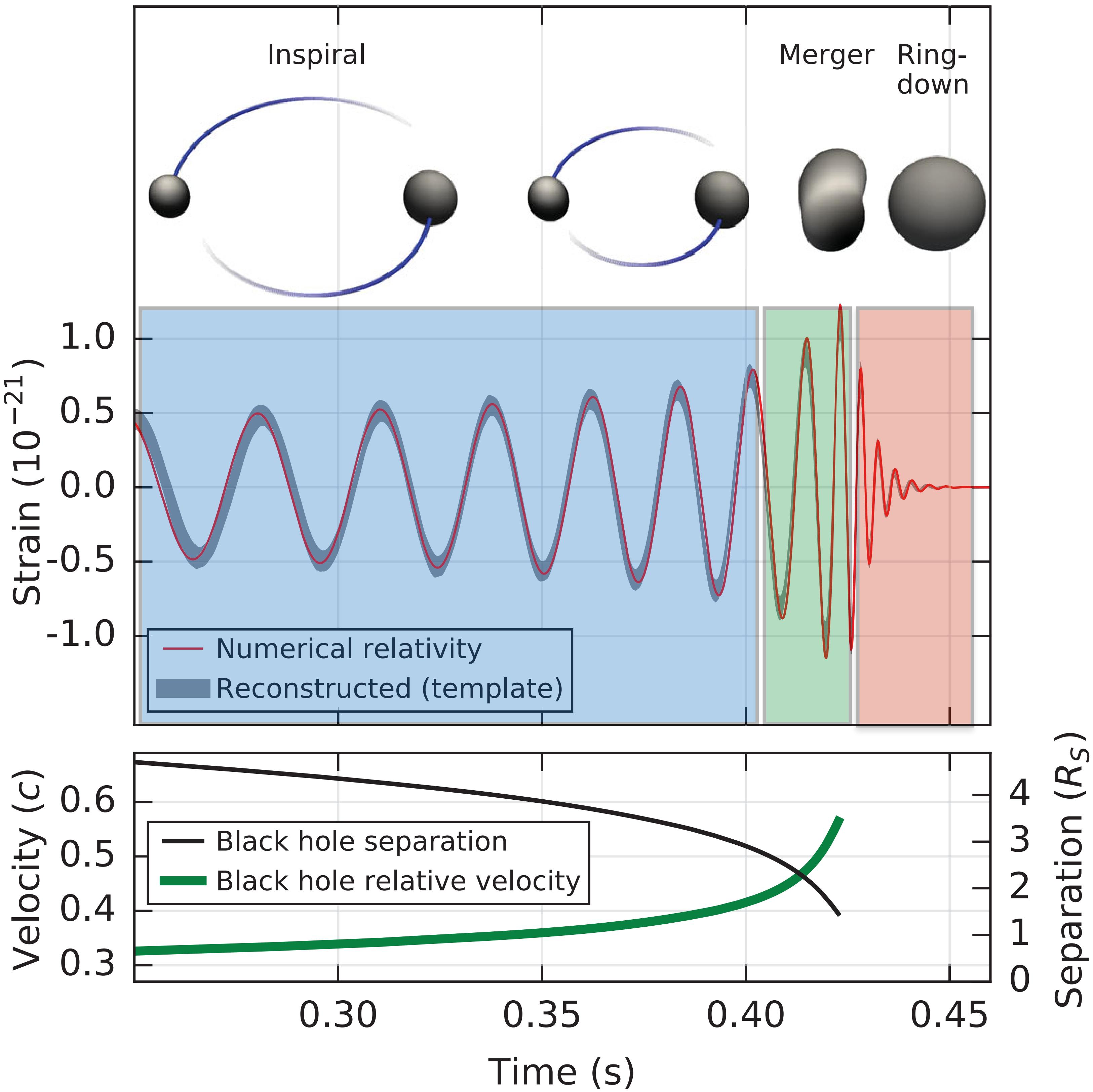




PRL 116, 061102 (2016)



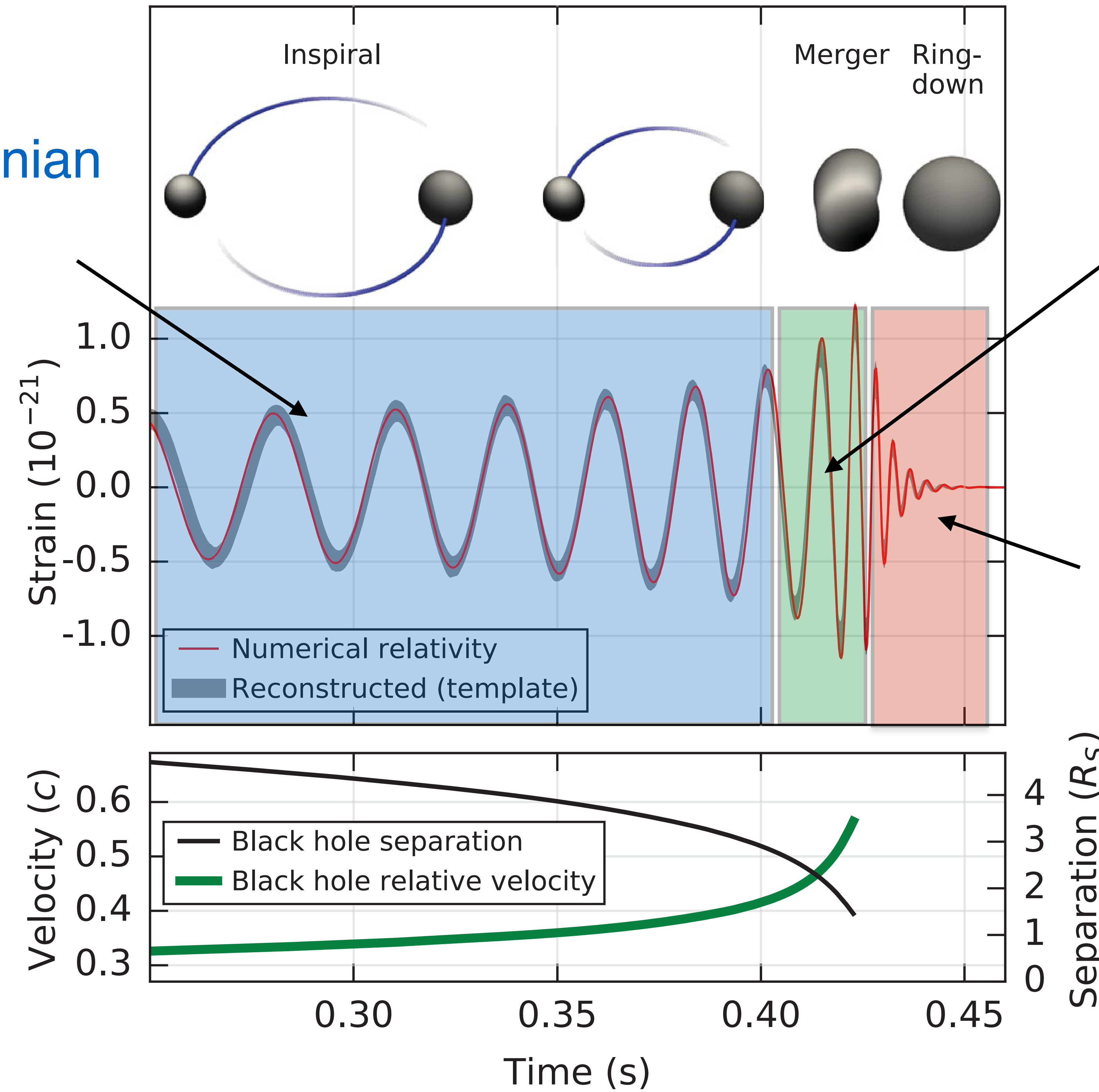
PRL 116, 061102 (2016)



PRL 116, 061102 (2016)

Post-Newtonian theory

Numerical relativity



BH perturbations QNM

PRL 116, 061102 (2016)

Post-Newtonian theory

- Perturbative expansion of relativistic effects
- 1 PN $\rightarrow \left(\frac{v}{c}\right)^2$
- More and more difficulties appear as we go to higher orders

$\boxed{\text{Blanchet-Damour-Iyer formalism}}$

Blanchet-Damour-Iyer formalism

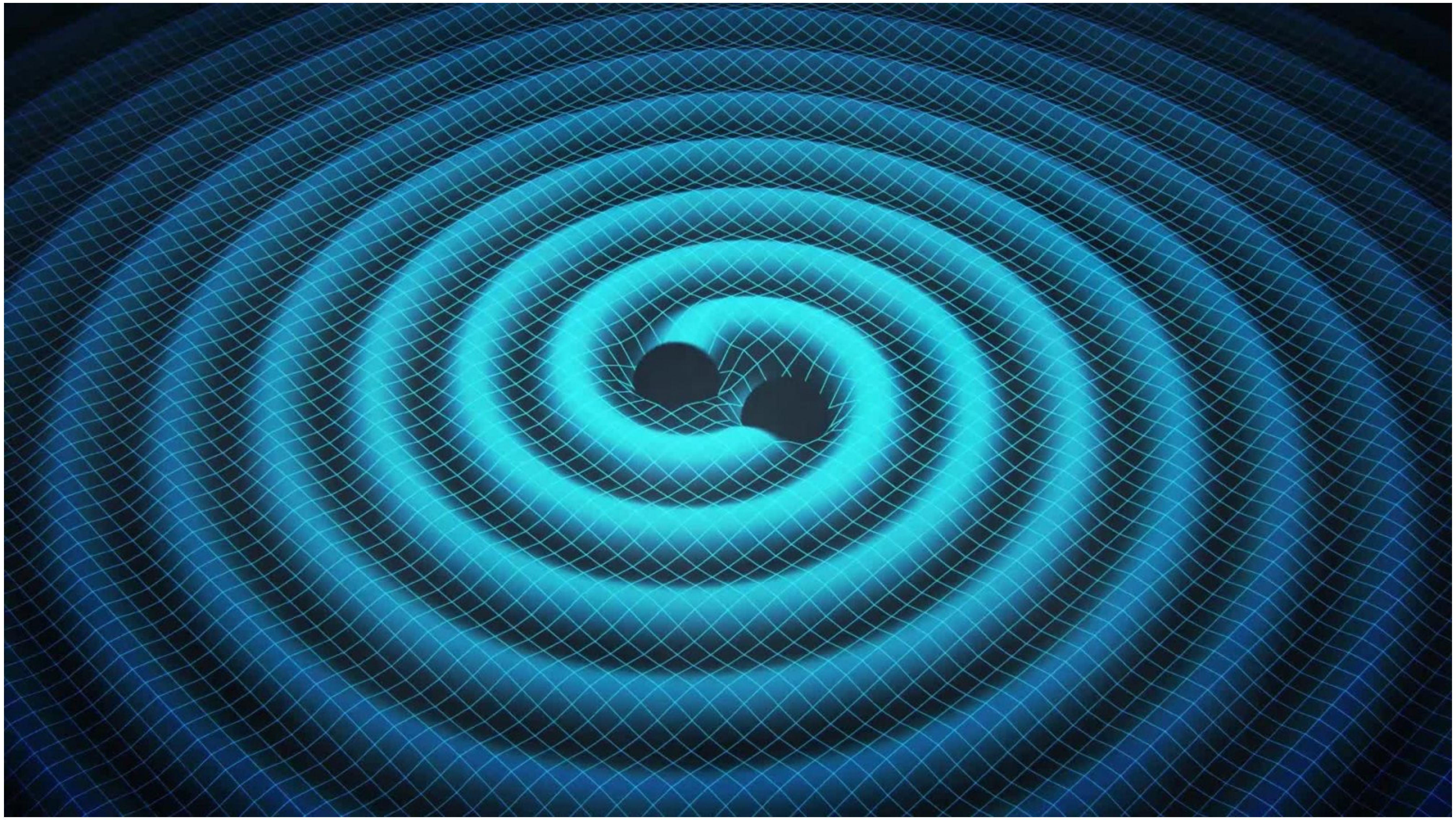


figure: www.virgo-gw.eu

Blanchet-Damour-Iyer formalism

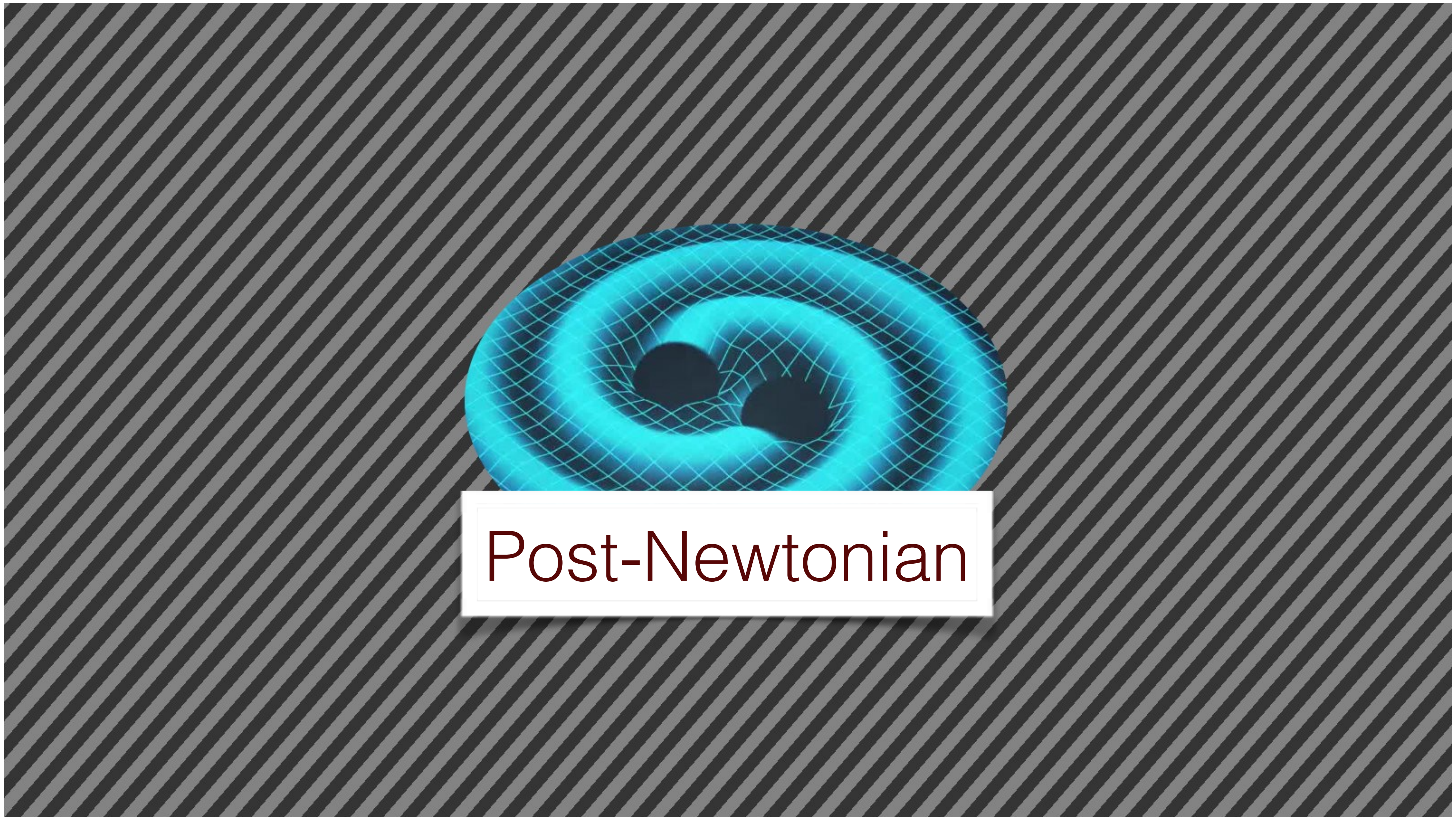


figure: www.virgo-gw.eu

Blanchet-Damour-Iyer formalism

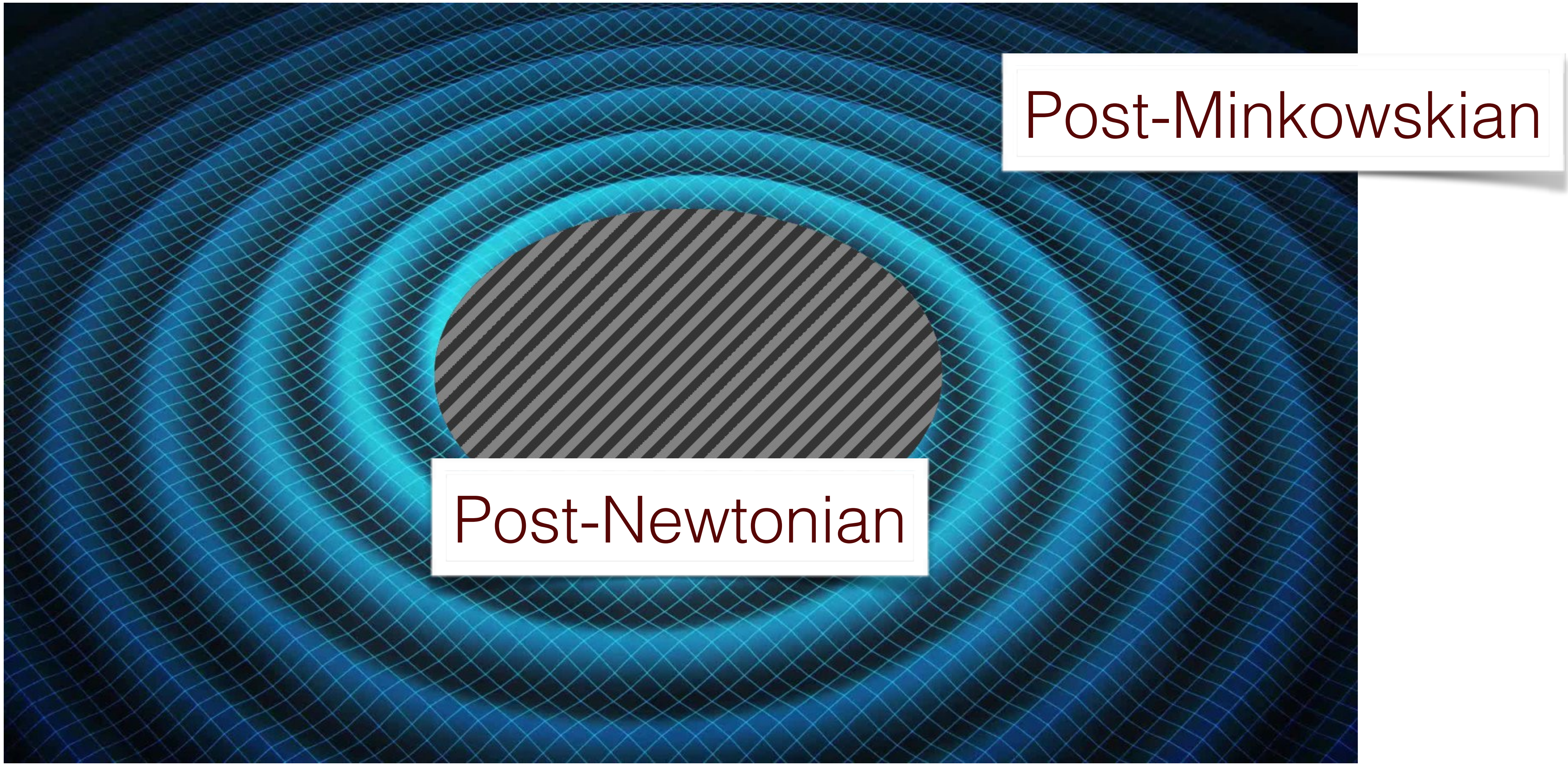


figure: www.virgo-gw.eu

Blanchet-Damour-Iyer formalism

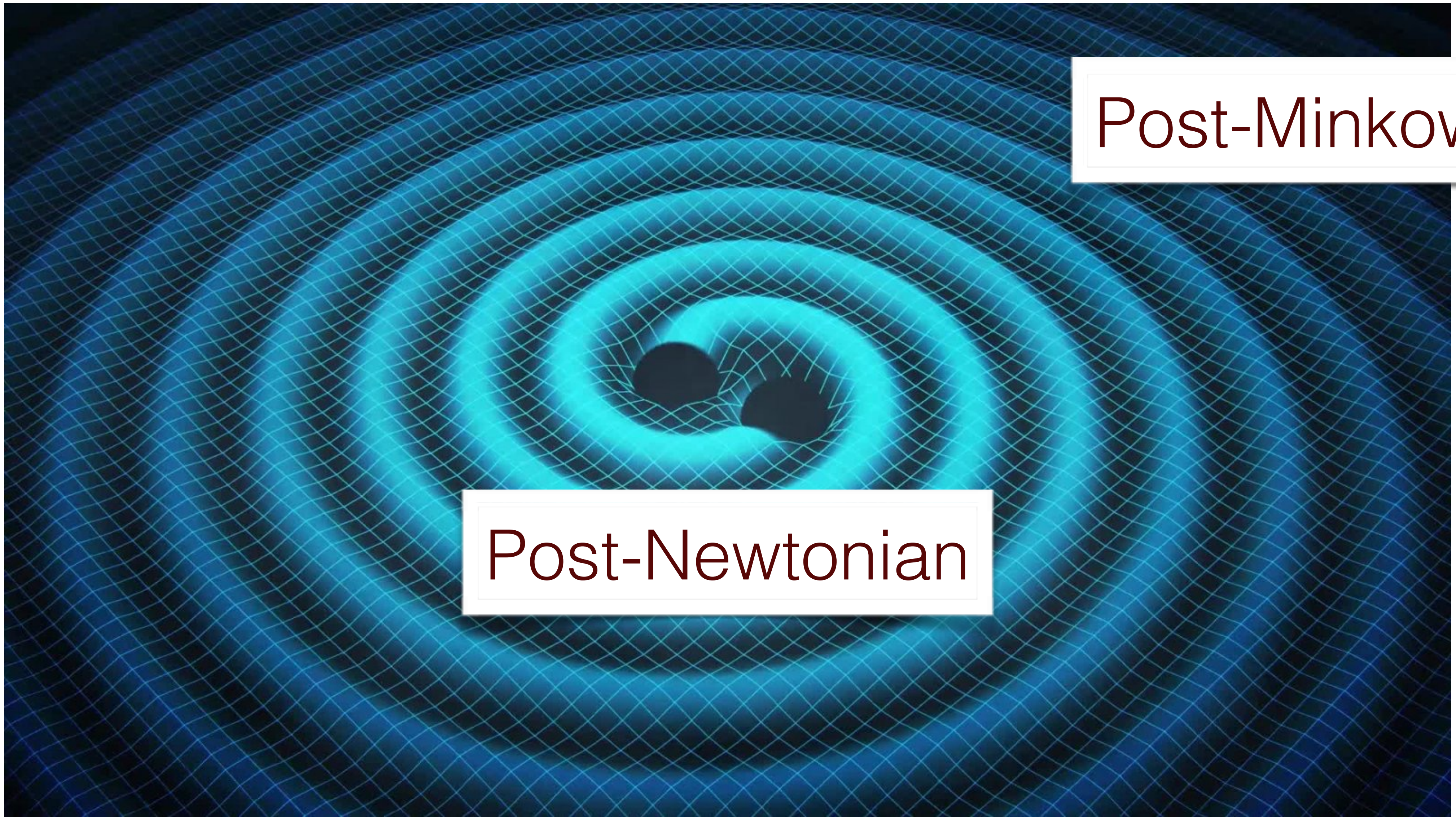


figure: www.virgo-gw.eu

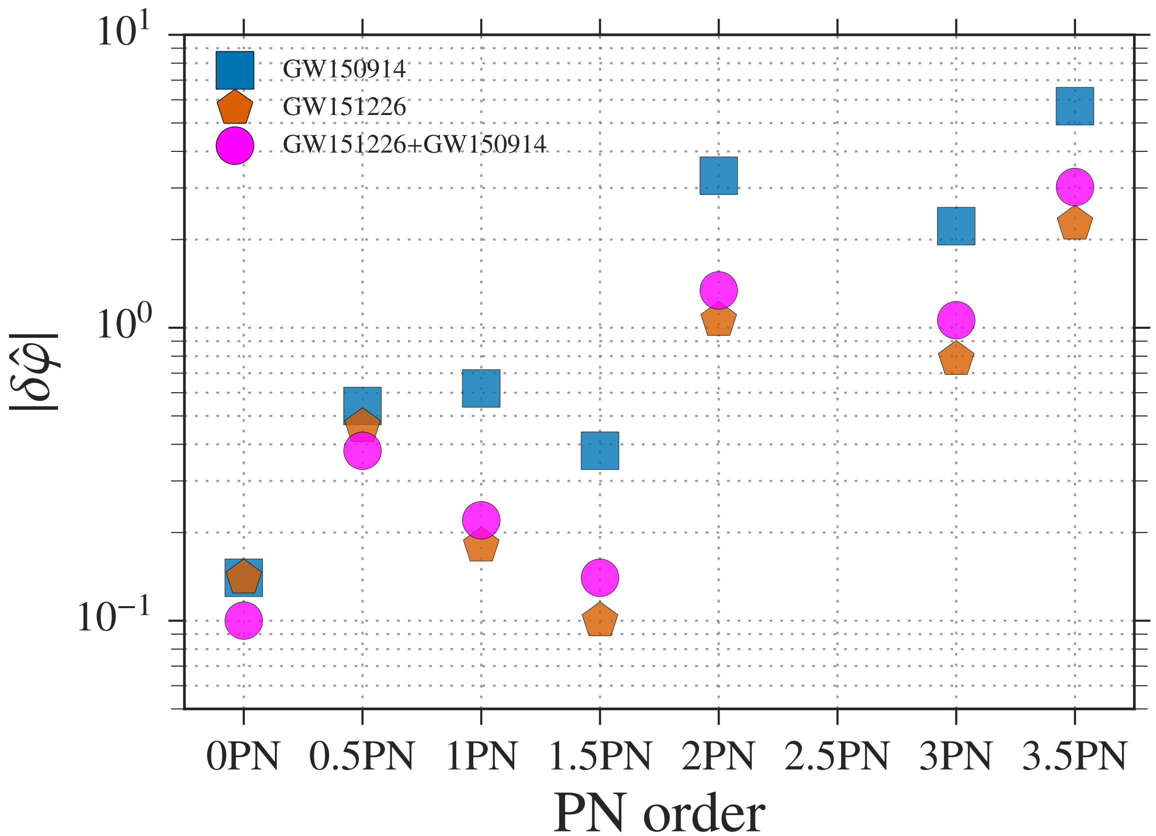
Once the **equations of motions** and the **flux** is known at n-PN:

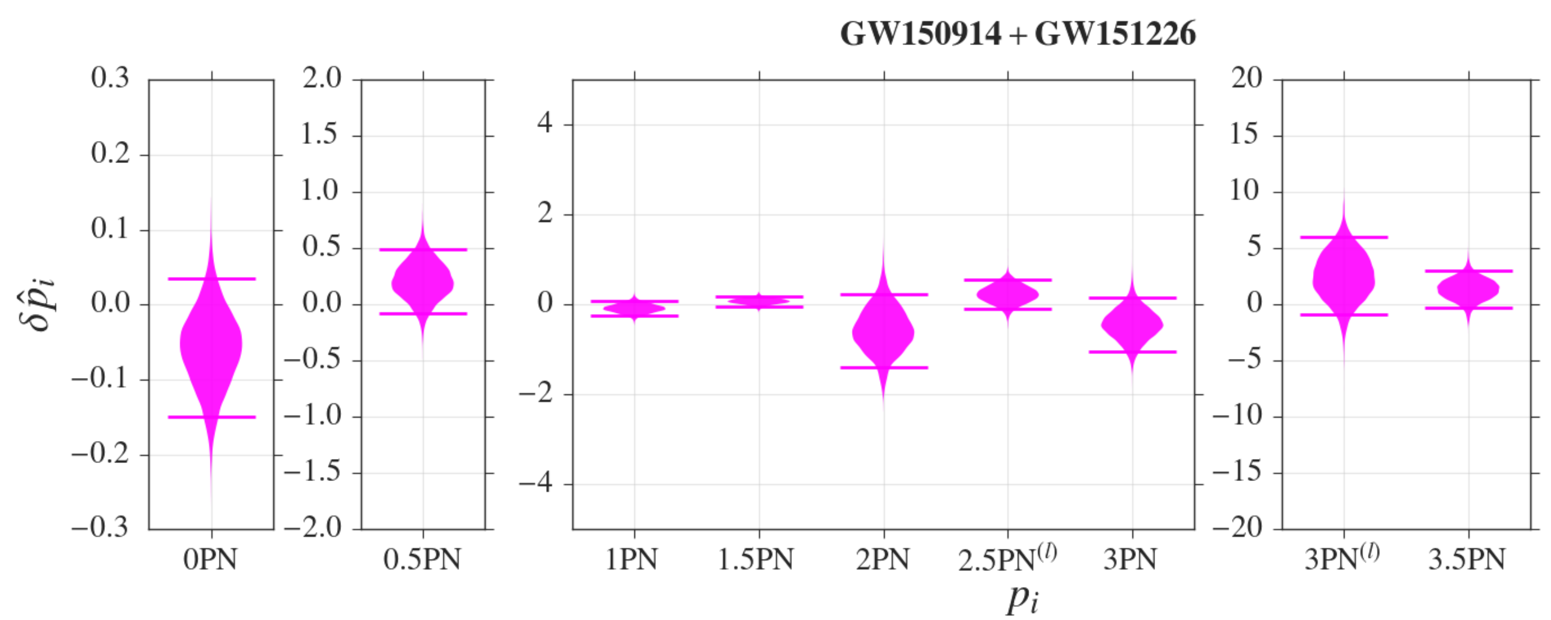
Once the **equations of motions** and the **flux** is known at n-PN:

$$\frac{\mathrm{d}E_{\mathrm{n}}}{\mathrm{d}t} = \mathcal{F}_{\mathrm{n}} \quad \Rightarrow \quad \phi_{\mathrm{n}} = f_{\mathrm{n}}(x)$$

Once the **equations of motions** and the **flux** is known at n-PN:

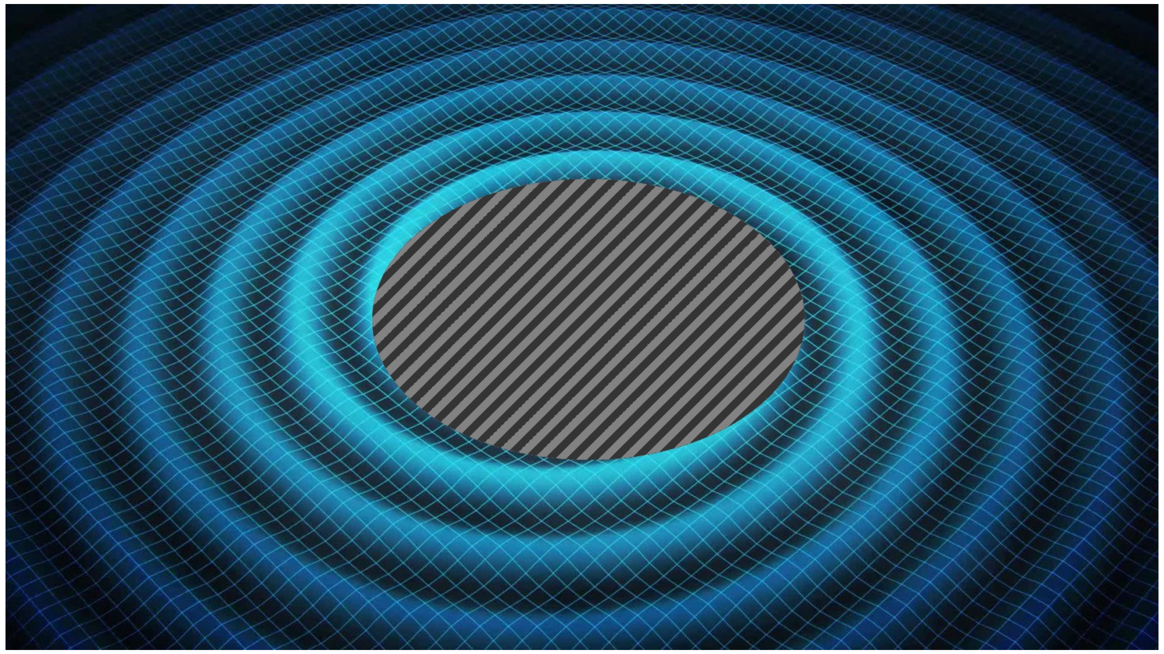
$$\frac{\mathrm{d}E_{\mathrm{n}}}{\mathrm{d}t} = \mathcal{F}_{\mathrm{n}} \quad \Rightarrow \quad \phi_{\mathrm{n}} = f_{\mathrm{n}}(x)$$





LIGO Scientific and Virgo collaboration arxiv:1606.04856

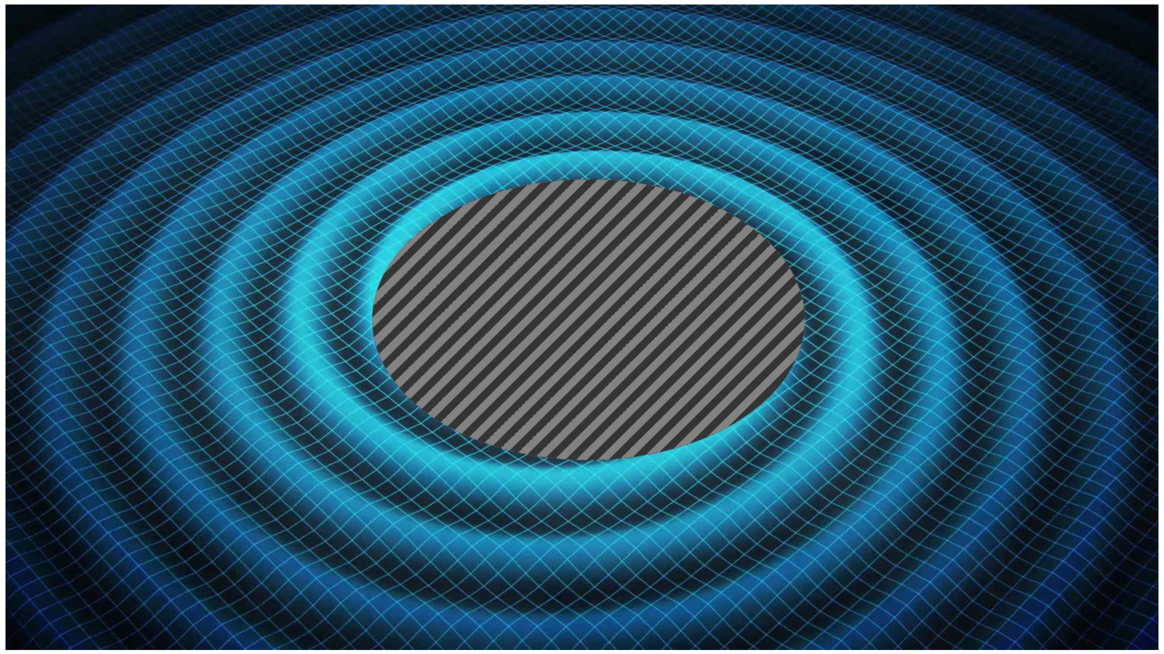
The MPM algorithm



$$G_{\mu\nu}(g_{\alpha\beta}, \partial g_{\alpha\beta}, \partial^2 g_{\alpha\beta}) = 0$$

$$h^{\mu\nu} \equiv \sqrt{-g} g^{\mu\nu} - \eta^{\mu\nu} = \mathcal{G} h^{1\mu\nu} + \mathcal{G}^2 h^{2\mu\nu} + \ldots$$

The MPM algorithm

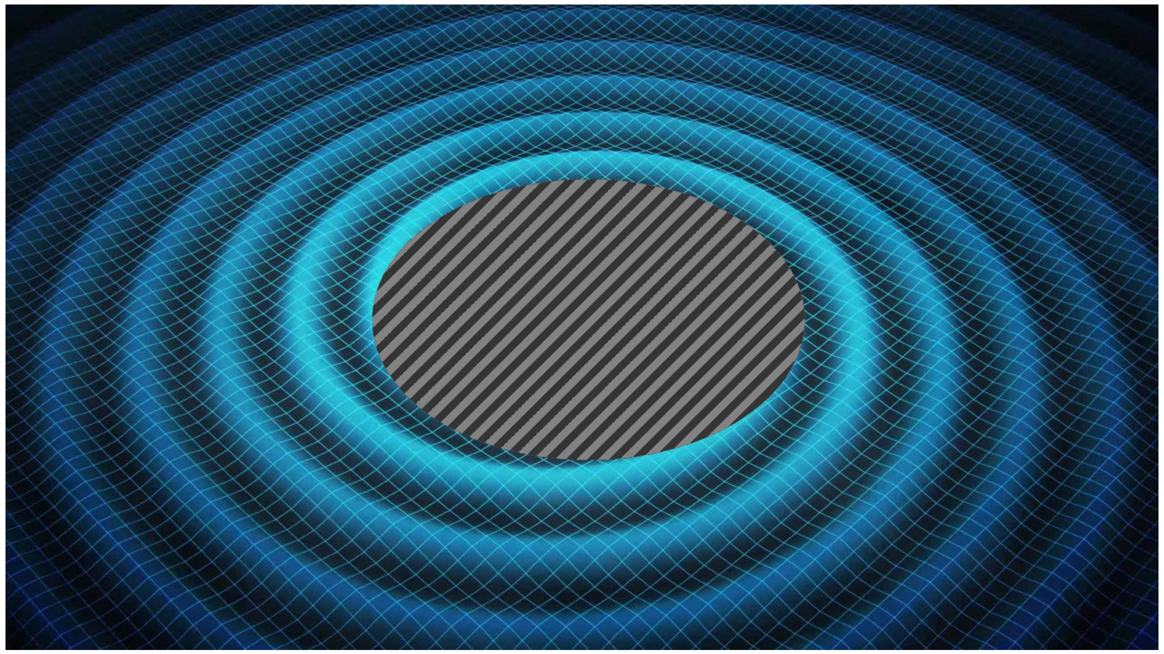


$$G_{\mu\nu}(g_{\alpha\beta}, \partial g_{\alpha\beta}, \partial^2 g_{\alpha\beta}) = 0$$

$$h^{\mu\nu} \equiv \sqrt{-g} g^{\mu\nu} - \eta^{\mu\nu} = \mathcal{G} h^{1\mu\nu} + \mathcal{G}^2 h^{2\mu\nu} + \dots$$

$$\begin{cases} \Box h^i_{\mu\nu} = \Lambda(h^1, \dots, h^{i-1}) \\ \partial^\mu h^i_{\mu\nu} = 0 \end{cases}$$

First issue: regularization

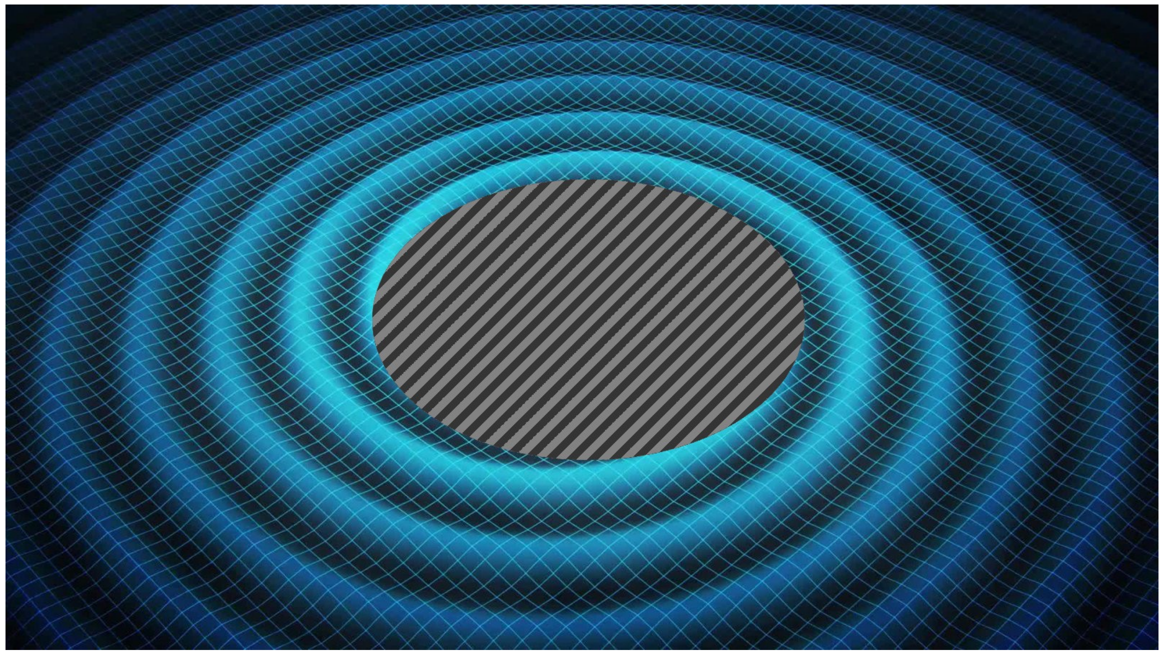


$$\begin{cases} \Box h^i_{\mu\nu} = \Lambda(h^1, \dots, h^{i-1}) \\ \partial^\mu h^i_{\mu\nu} = 0 \end{cases}$$

$$\Box^{-1}\Lambda(x,t) = \int \mathrm{d}^3x' \, \frac{\Lambda(x', t - \mid x - x' \mid)}{\mid x - x' \mid}$$

Issue: $\Lambda \sim_{r\to 0} \frac{1}{r^k}, \; k \geq 3$

First issue: regularization



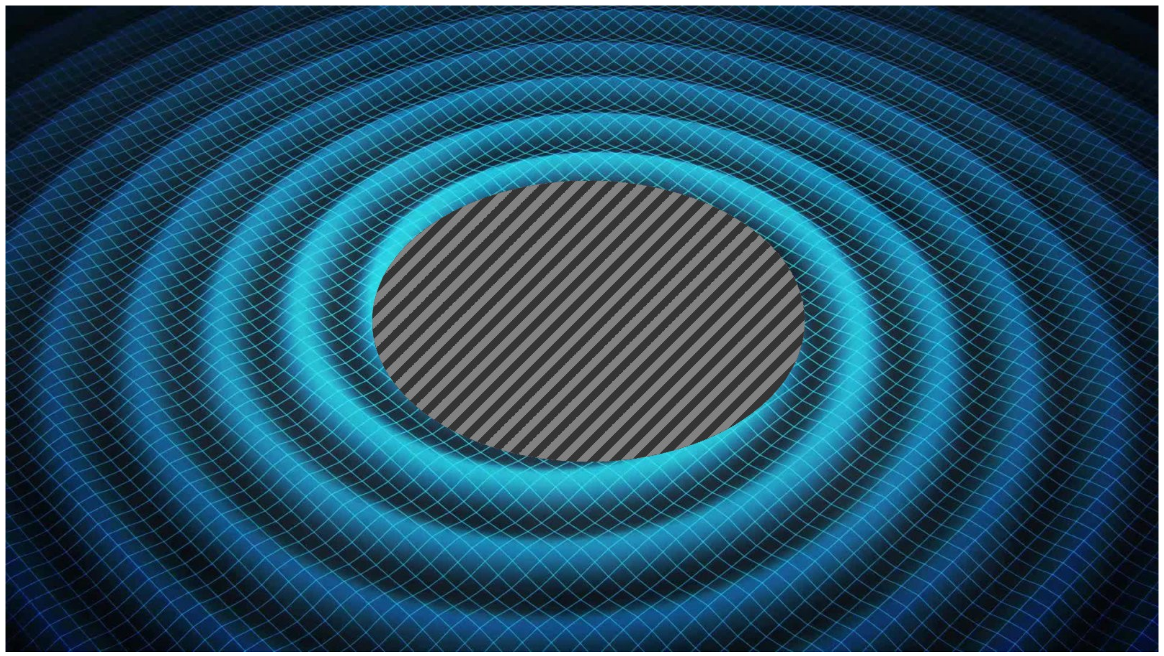
$$\begin{cases} \Box h^i_{\mu\nu} = \Lambda(h^1, \ldots, h^{i-1}) \\ \partial^\mu h^i_{\mu\nu} = 0 \end{cases}$$

$$\Box^{-1}\Lambda(x,t) = \int \mathrm{d}^3 x' \frac{\Lambda(x', t - \mid x - x' \mid)}{\mid x - x' \mid}$$

Issue: $\Lambda \sim_{r\to 0} \frac{1}{r^k}, \; k \geq 3$

$$\mathrm{FP}_{B=0}\Box^{-1}\left[\left(\frac{r}{r_0}\right)^B \Lambda\right]$$

First issue: regularization



$$\begin{cases} \Box h^i_{\mu\nu} = \Lambda(h^1, \ldots, h^{i-1}) \\ \partial^\mu h^i_{\mu\nu} = 0 \end{cases}$$

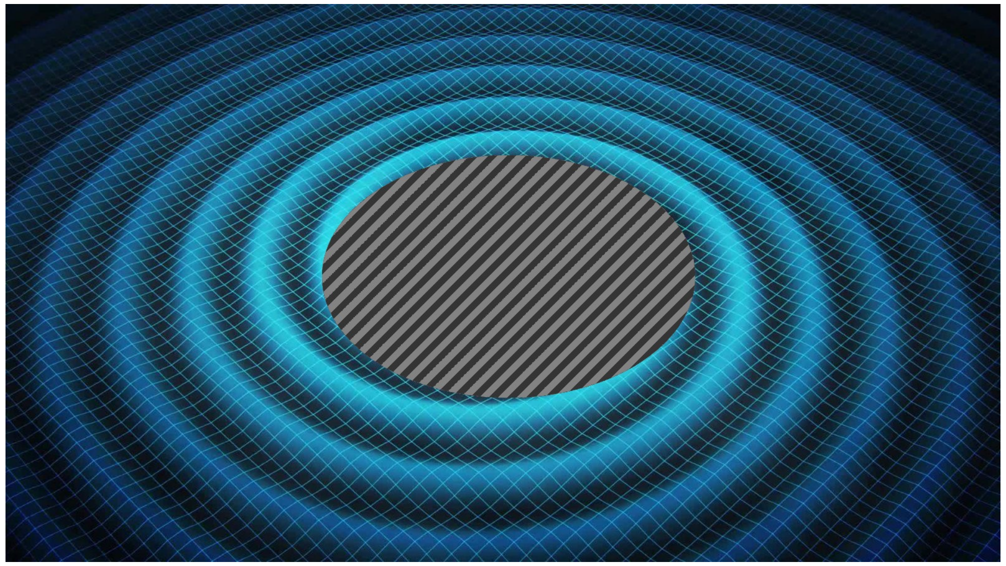
$$\Box^{-1}\Lambda(x,t) = \int \mathrm{d}^3x' \frac{\Lambda(x', t - \mid x - x' \mid)}{\mid x - x' \mid}$$

Issue: $\Lambda \sim_{r\to 0} \frac{1}{r^k}, \; k \geq 3$

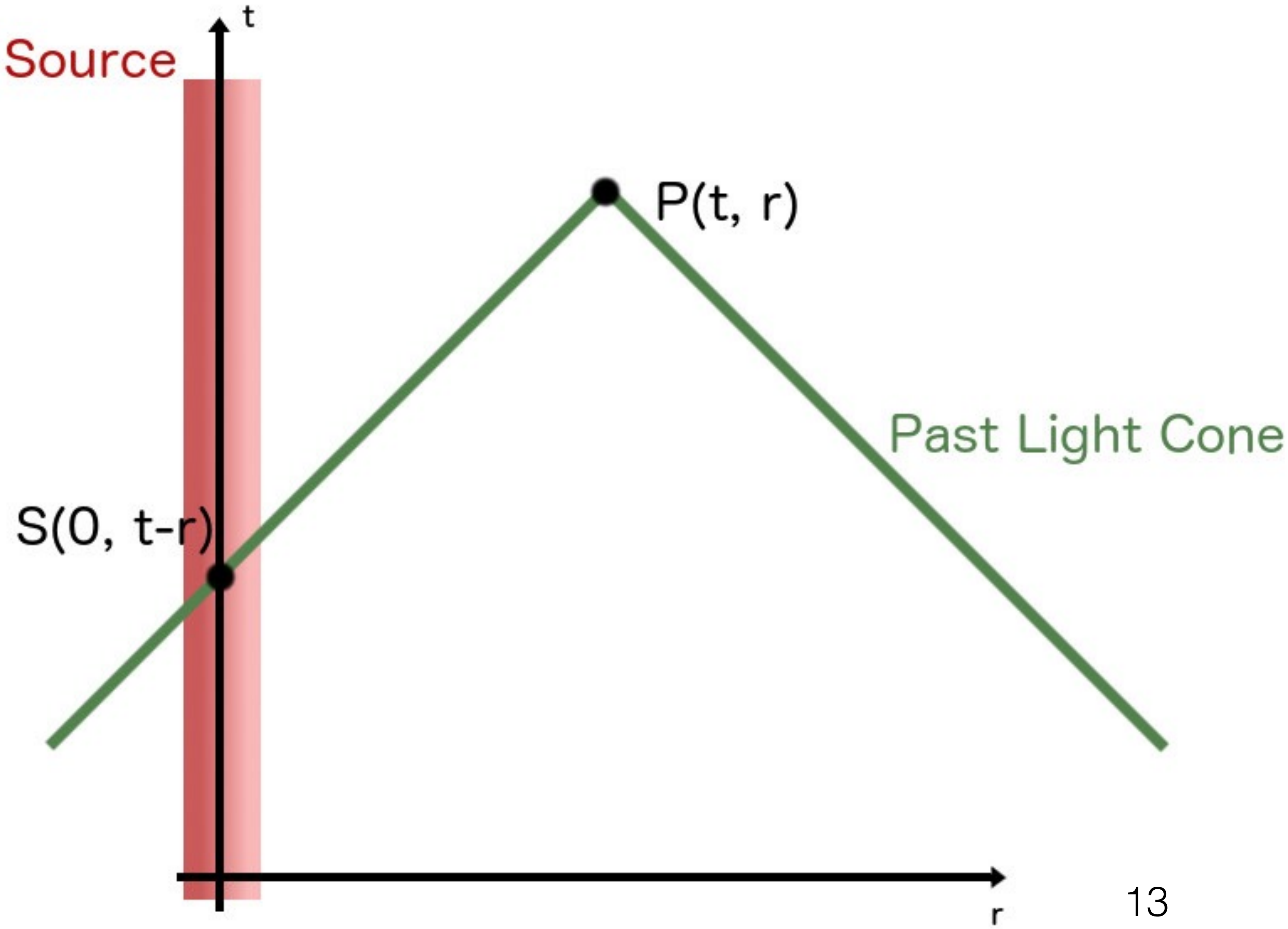
$$\mathrm{FP}_{B=0}\Box^{-1}\left[\left(\frac{r}{r_0}\right)^B \Lambda\right]$$

$$\mathrm{FP}_{B=0}\left[\sum_{k\geq -k_0} g_k B^k\right] \equiv g_0$$

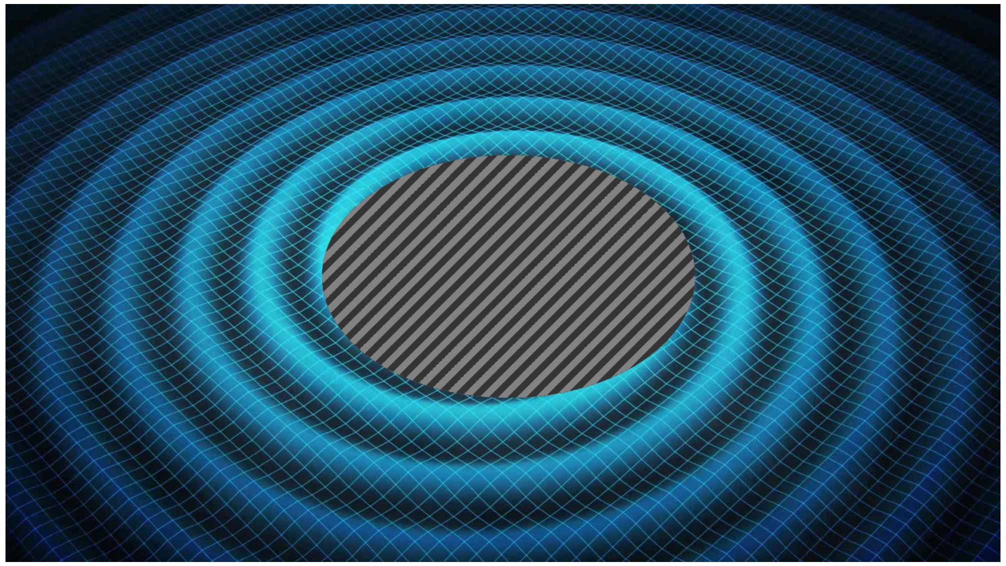
Second issue: tails



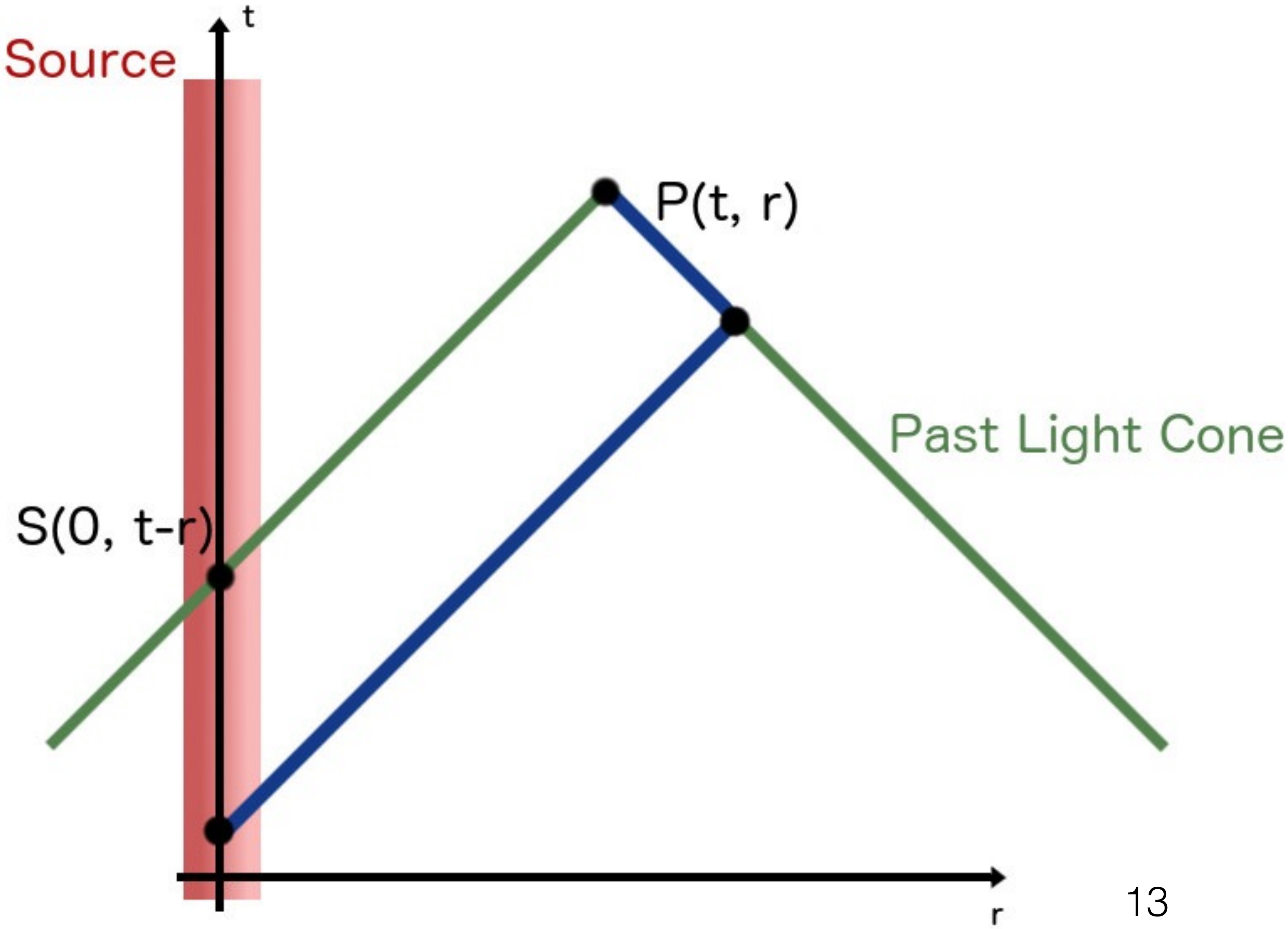
$$
\begin{cases} \Box h^i_{\mu\nu} = \Lambda(h^1, \ldots, h^{i-1}) \\ \partial^\mu h^i_{\mu\nu} = 0 \end{cases}
$$



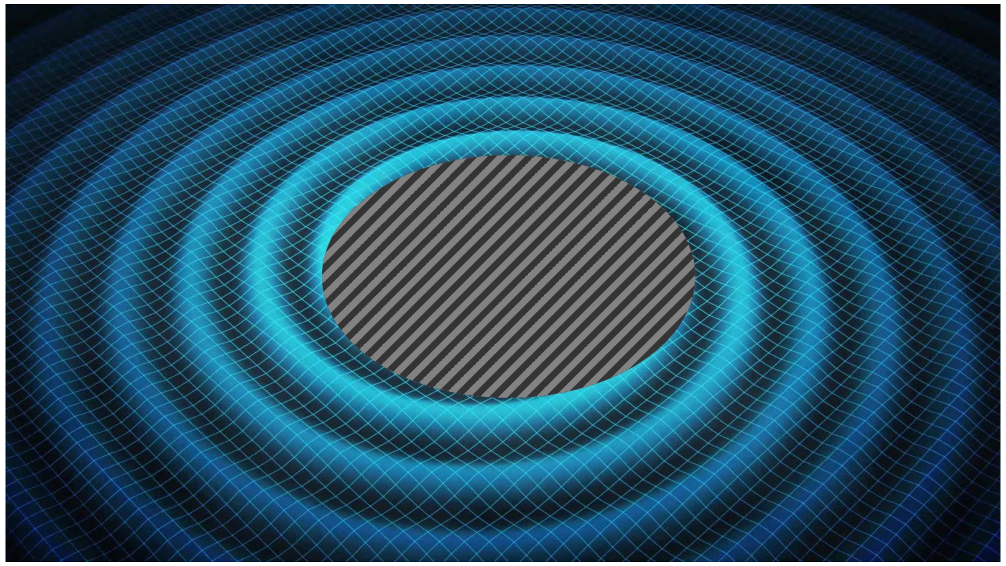
Second issue: tails



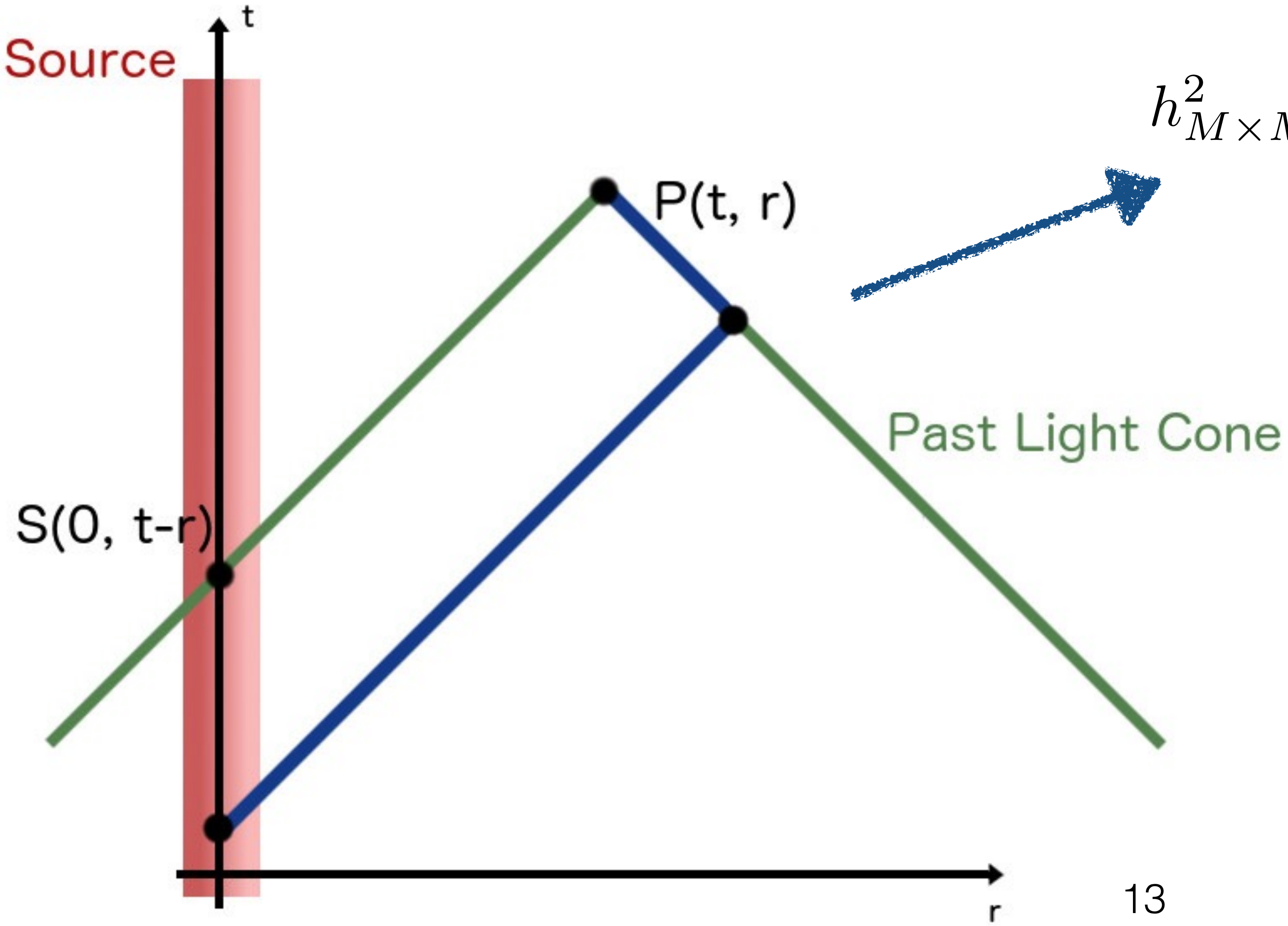
$$\begin{cases} \Box h^i_{\mu\nu} = \Lambda(h^1, \ldots, h^{i-1}) \\ \partial^\mu h^i_{\mu\nu} = 0 \end{cases}$$



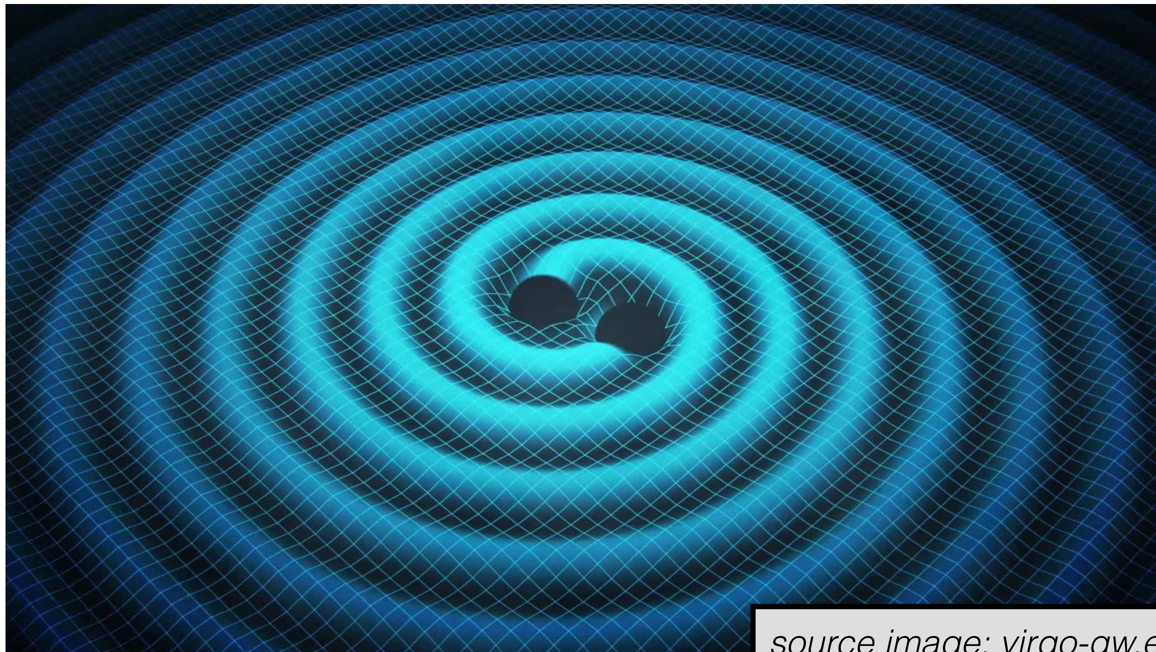
Second issue: tails



$$\begin{cases} \Box h^i_{\mu\nu} = \Lambda(h^1, \ldots, h^{i-1}) \\ \partial^\mu h^i_{\mu\nu} = 0 \end{cases}$$



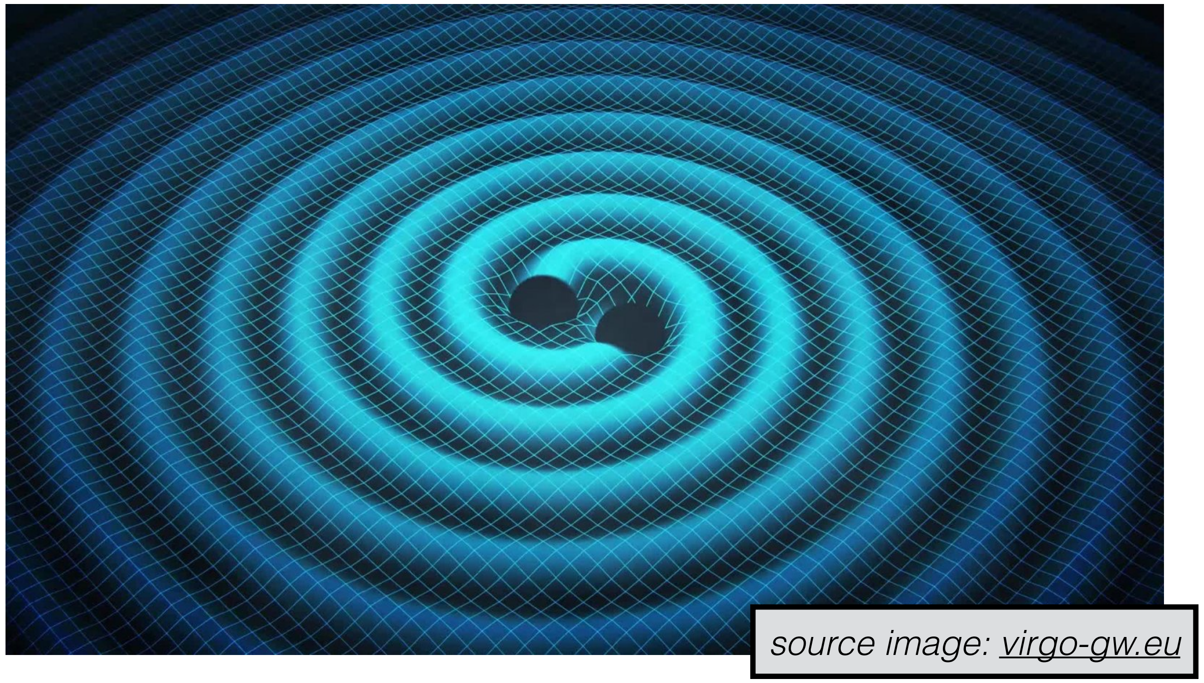
$$h^2_{M\times M_{ij}}(t,r) \sim M \int_0^\infty \mathrm{d}\tau M_{ij}(t-r-\tau)\mathcal{Q}(\tau)$$



source image: virgo-gw.eu

$$\nu = \frac{M_1 M_2}{(M_1 + M_2)^2}$$

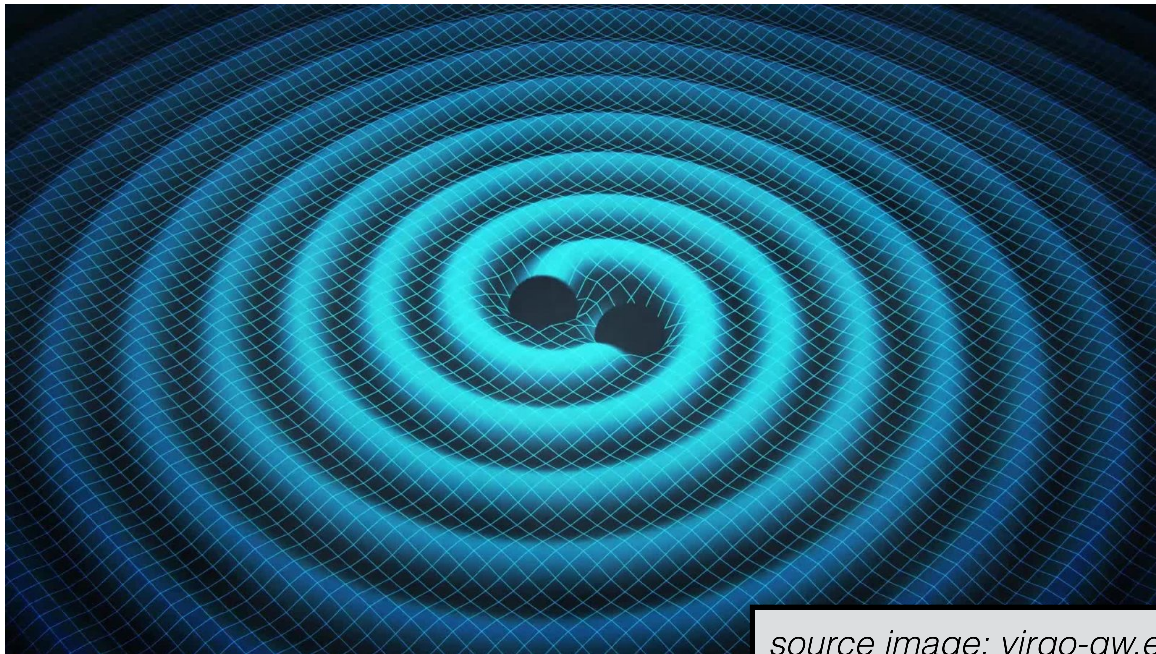
$$x = \left(\frac{G M_{tot} \Omega}{c^3}\right)^{2/3} = \mathcal{O}\left(\frac{1}{c^2}\right)$$



$$\nu = \frac{M_1 M_2}{(M_1+M_2)^2}$$

$$x = \left(\frac{G M_{tot}\Omega}{c^3}\right)^{2/3} = \mathcal{O}\left(\frac{1}{c^2}\right)$$

$$\begin{aligned}\phi = -\frac{x^{-5/2}}{32\nu}\Bigg\{1 &+ \left(\frac{3715}{1008}+\frac{55}{12}\nu\right)x - 10\pi x^{3/2} \\ &+ \left(\frac{15293365}{1016064}+\frac{27145}{1008}\nu+\frac{3085}{144}\nu^2\right)x^2 + \left(\frac{38645}{1344}-\frac{65}{16}\nu\right)\pi x^{5/2}\ln\left(\frac{x}{x_0}\right) \\ &+ \Bigg[\frac{12348611926451}{18776862720} - \frac{160}{3}\pi^2 - \frac{1712}{21}\gamma_{\rm E} - \frac{856}{21}\ln(16\,x) \\ &\qquad + \left(-\frac{15737765635}{12192768}+\frac{2255}{48}\pi^2\right)\nu + \frac{76055}{6912}\nu^2 - \frac{127825}{5184}\nu^3\Bigg]x^3 \\ &+ \left(\frac{77096675}{2032128}+\frac{378515}{12096}\nu-\frac{74045}{6048}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\Bigg\},\end{aligned}$$



source image: virgo-gw.eu

$$\nu = \frac{M_1 M_2}{(M_1+M_2)^2}$$

$$x = \left(\frac{G M_{tot}\Omega}{c^3}\right)^{2/3} = \mathcal{O}\left(\frac{1}{c^2}\right)$$

$$\begin{aligned}\phi = -\frac{x^{-5/2}}{32\nu}\Bigg\{ & 1 + \left(\frac{3715}{1008}+\frac{55}{12}\nu\right)x - 10\pi x^{3/2} \\ & + \left(\frac{15293365}{1016064}+\frac{27145}{1008}\nu+\frac{3085}{144}\nu^2\right)x^2 + \left(\frac{38645}{1344}-\frac{65}{16}\nu\right)\pi x^{5/2}\ln\left(\frac{x}{x_0}\right) \\ & + \left[\frac{12348611926451}{18776862720}-\frac{160}{3}\pi^2-\frac{1712}{21}\gamma_{\mathrm{E}}-\frac{856}{21}\ln(16\,x)\right. \\ & \qquad \left. + \left(-\frac{15737765635}{12192768}+\frac{2255}{48}\pi^2\right)\nu + \frac{76055}{6912}\nu^2 - \frac{127825}{5184}\nu^3\right]x^3 \\ & + \left(\frac{77096675}{2032128}+\frac{378515}{12096}\nu-\frac{74045}{6048}\nu^2\right)\pi x^{7/2} + \mathcal{O}\left(\frac{1}{c^8}\right)\Bigg\},\end{aligned}$$

4.5PN results: Marchand et al, 2016 (accepted to **CQG**)

Projects for 2nd and 3rd year

Projects for 2nd and 3rd year

- Working on the near-zone physics at 4PN

Projects for 2nd and 3rd year

- Working on the near-zone physics at 4PN

- Studying the Vainshtein mechanism in some class of modified gravity theories.

Thank you