Gravitational waves from compact binaries detection and parameter estimation

Yann Bouffanais and Edward K. Porter

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First detection of gravitational waves

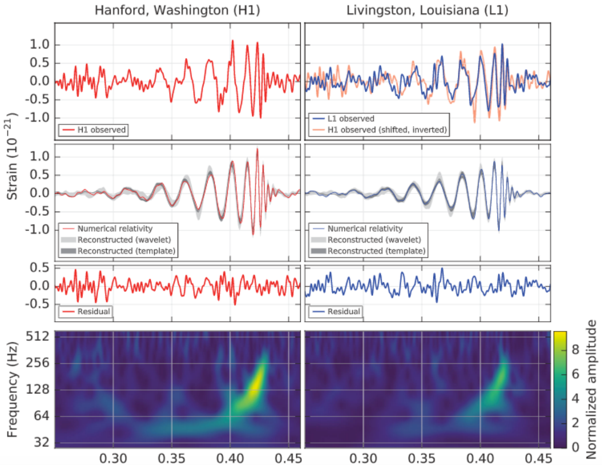


Figure: Gravitational wave signal from a binary black hole merger observed by the two LIGO detectors during O1 (GW150914).

Spectrum of gravitation waves: sources and detectors

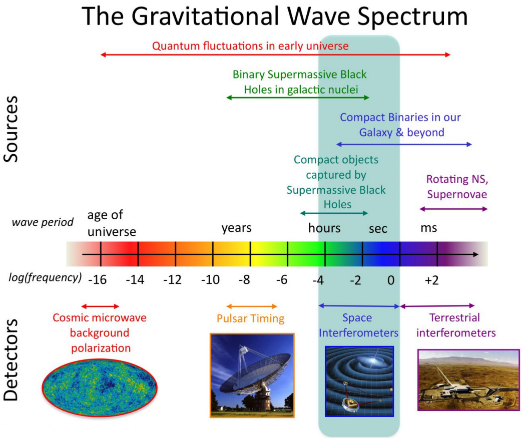
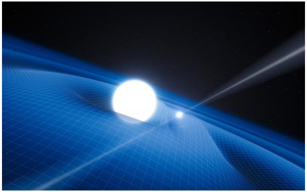


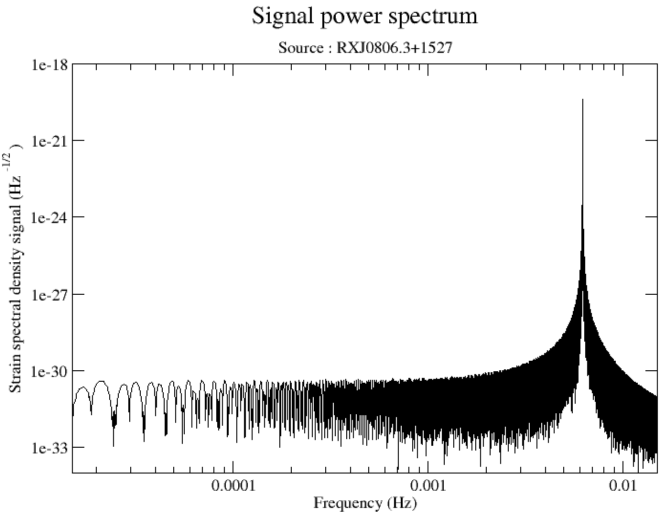
Figure: Gravitational waves spectrum along with sources and detectors

Compact galactic binaries

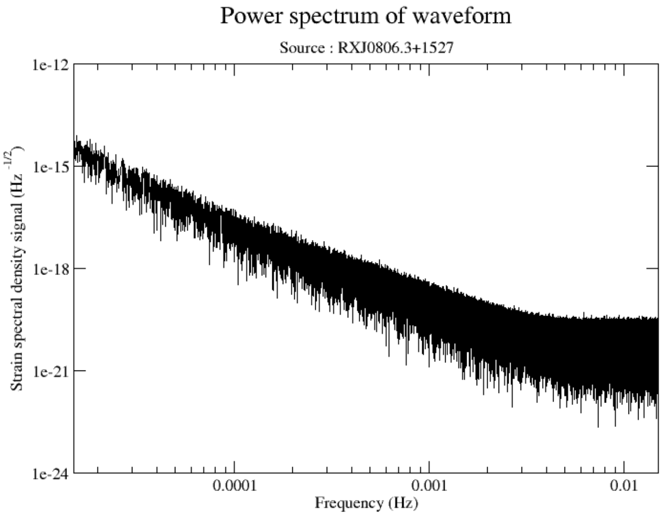
- Stellar compact galactic binaries: white dwarfs, neutron stars, stellar mass black holes.
- Interesting astrophysical objects with a variety of scenarios : Roche lobe overflow, stable/unstable mass transfer, nova, X-ray burster, type Ia supernovae, common envelope phase, coalescence
- Resolvable sources for both detectors
  - During orbital phase for eLISA
  - During end-life coalescence for LIGO/VIRGO



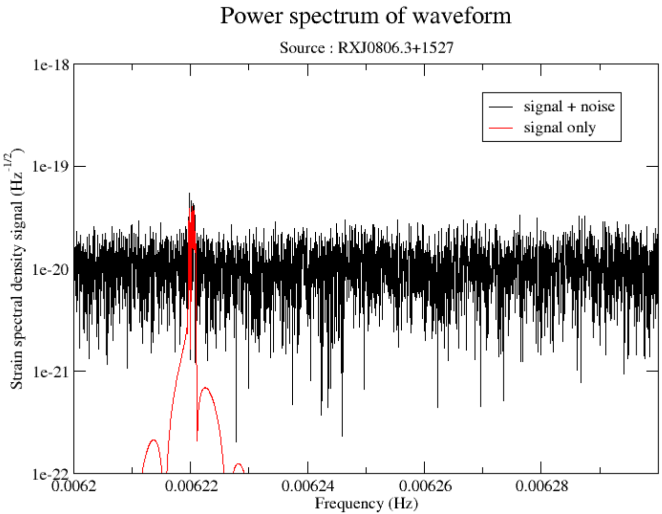
Wave Illustration - Signal Power spectrum



Wave Illustration - Signal + noise power spectrum



Wave Illustration - Signal + noise power spectrum



Search algorithm

- **Particle swarm optimization**: swarm-based algorithm whose dynamic mimics movement of swarm observed in Nature
- **Differential evolution**: population-based algorithm inspired by evolution and genetic laws
- **Markov Chain Monte Carlo**: stochastic Markovian sampler used essentially to sample posteriors
- **Uphill climber**: greedy criterion proposal Markov Chain Monte Carlo-like algorithm used for local exploration

Multi sources search

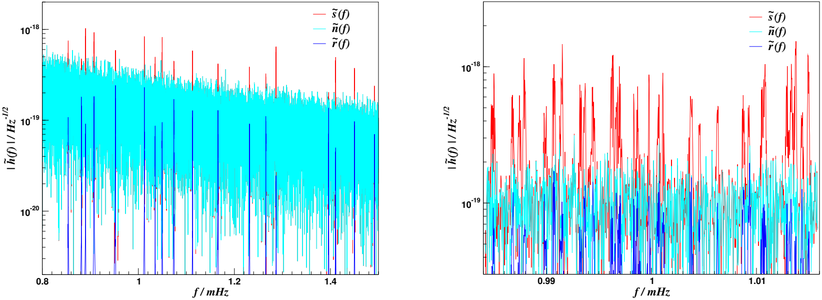


Figure: A plot of the power spectra for two injected data sets, no confusion (left) and mild confusion (right), along with the associated instrumental noises and found residuals (*Bouffanais and Porter, Phys. Rev. D 93, 064020 (2016)*)

Parameter estimation

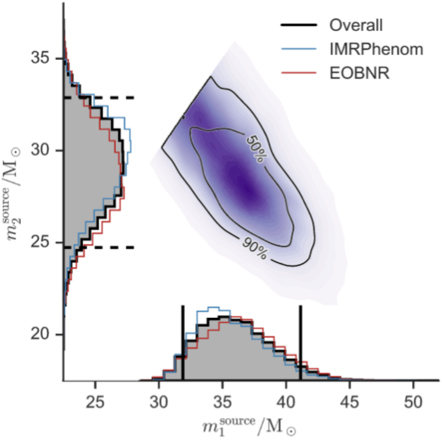


Figure: Posterior probability density functions for the source-frame component masses m_1 and m_2 for GW150914.

Parameter estimation in LIGO/VIRGO

- Bayesian techniques are used to estimate the probability density functions for the parameters of the source. Two algorithms currently integrated in LALInference:
  - Markov Chain Monte Carlo (MCMC)
  - Nested Sampling
- Crucial to have efficient and fast techniques for parameter estimation
  - More sources expected in O2 run
  - Electromagnetic follow-up

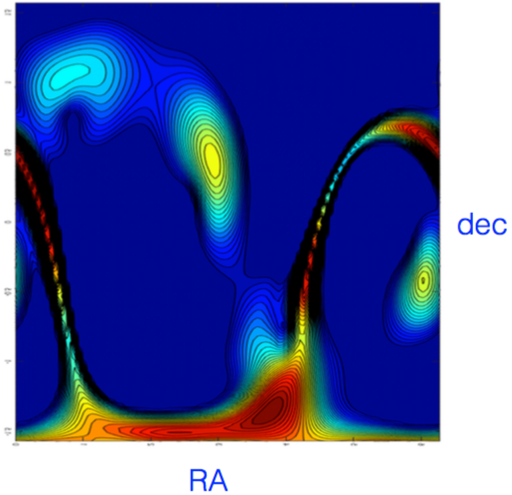
HMC

- Sampling method developed by Duane et al. in 1987
- The inverse posterior density can be thought of as a "gravitational potential"
- We can then consider the parameter values as state space variables q^μ and introduce a set of associated canonical momenta p^μ
- The Hamiltonian is then defined using a "mass matrix" $M_{\mu\nu}$ as

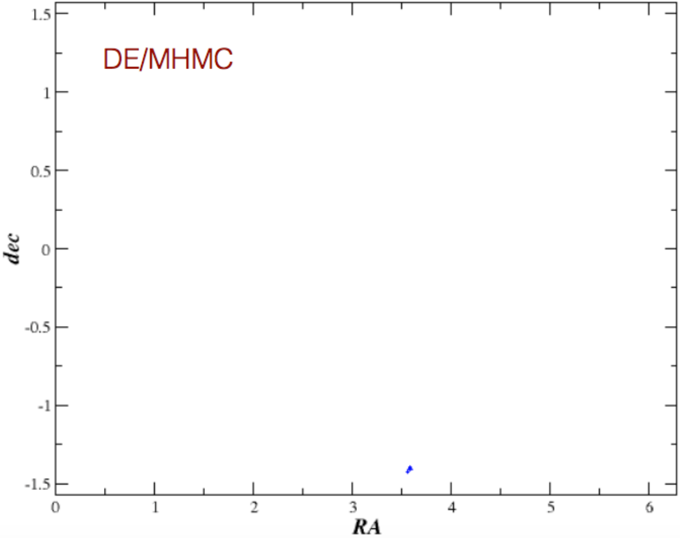
$$\mathcal{H}(q^\mu, p^\mu) = -\ln \mathcal{L}(q^\mu) + \frac{1}{2} M^{-1}_{\mu\nu} p^\mu p^\nu \quad (1)$$

- Use a leapfrog algorithm to solve Hamilton's equations with step size ϵ and length l

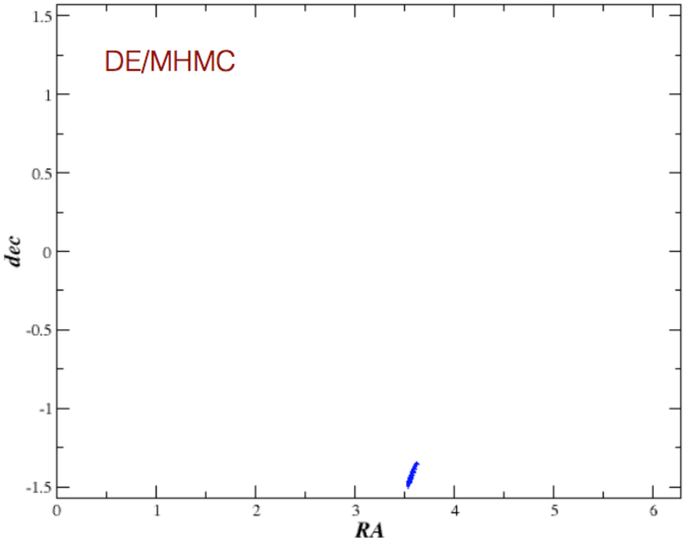
Example posteriors



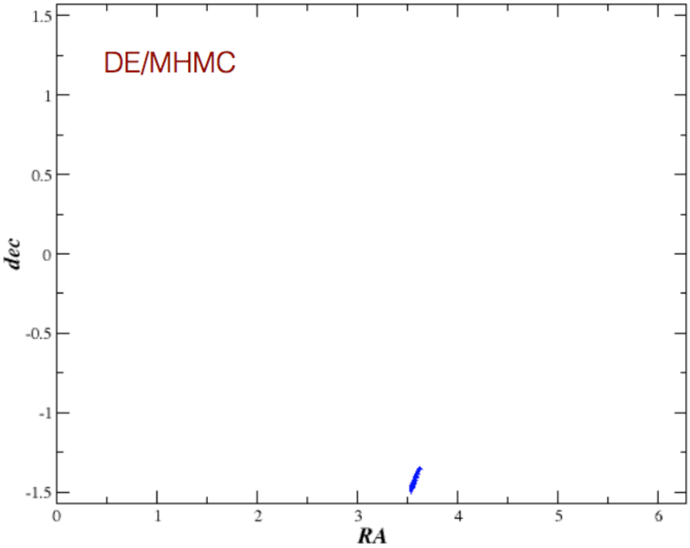
DEMC: 10^2 points, acceptance : 11%



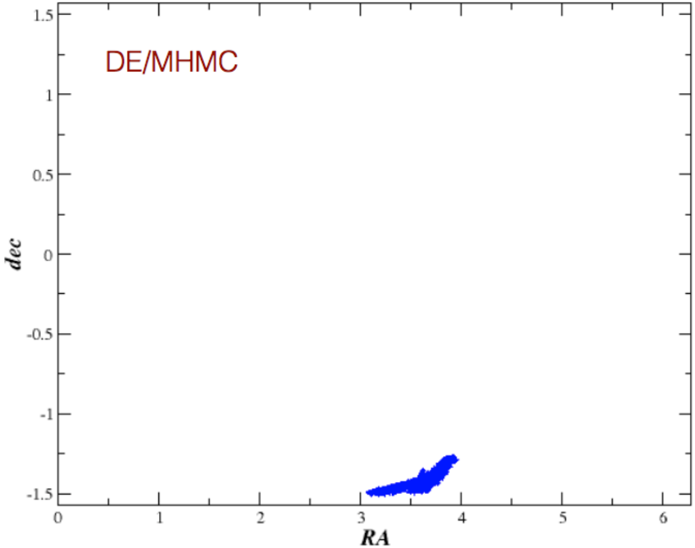
DEMC: 10^3 points, acceptance : 28%



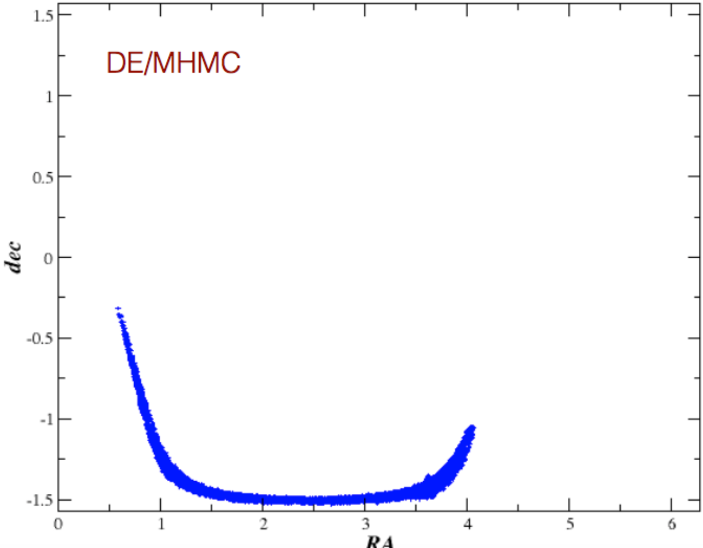
DEMC: 10^4 points, acceptance : 65%



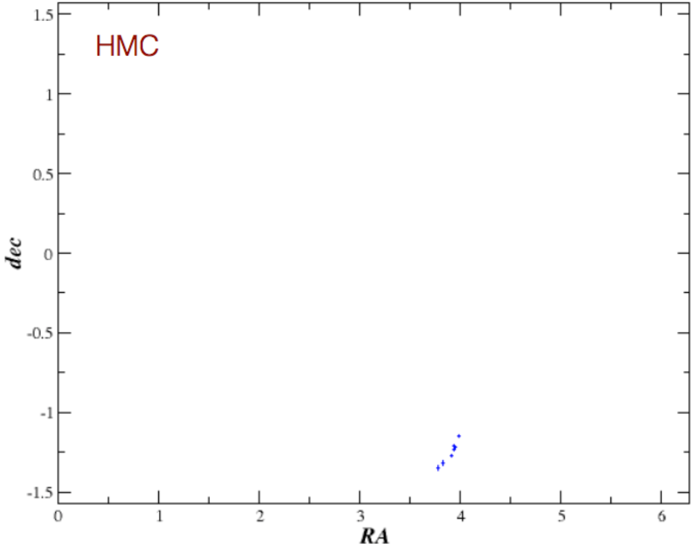
DEMC: 10^5 points, acceptance : 62%



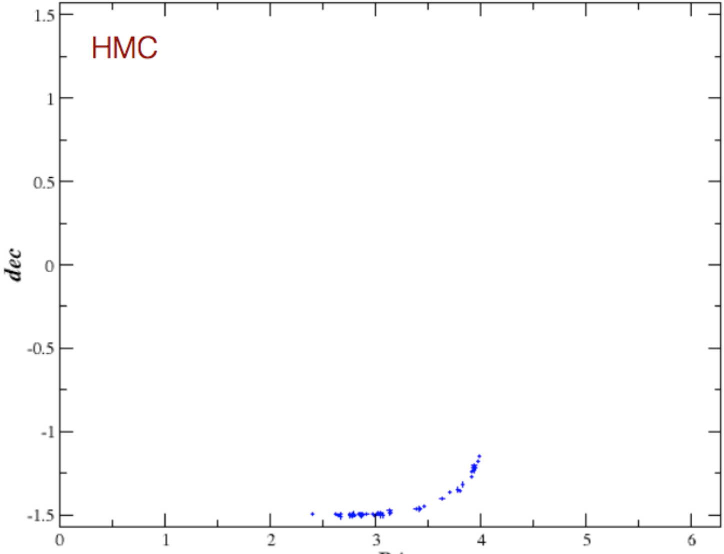
DEMC: 10^6 points, acceptance : 18%



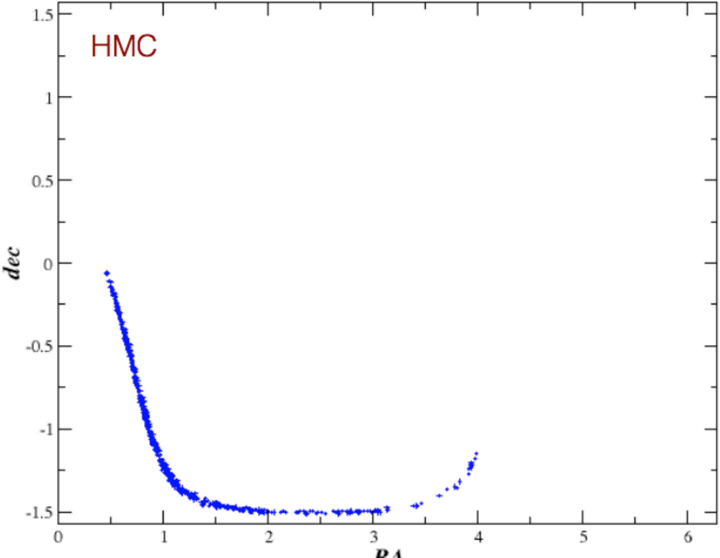
HMC: 10 points, acceptance : 70%



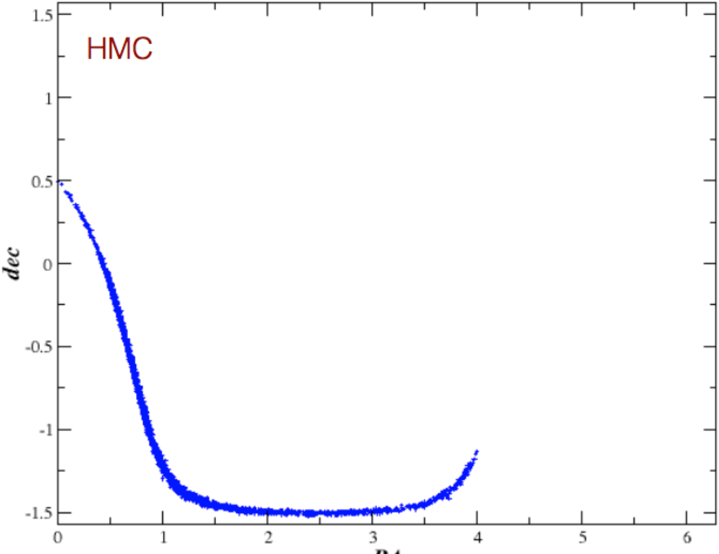
HMC: 10^2 points, acceptance : 55%



HMC: 10^3 points, acceptance : 71%



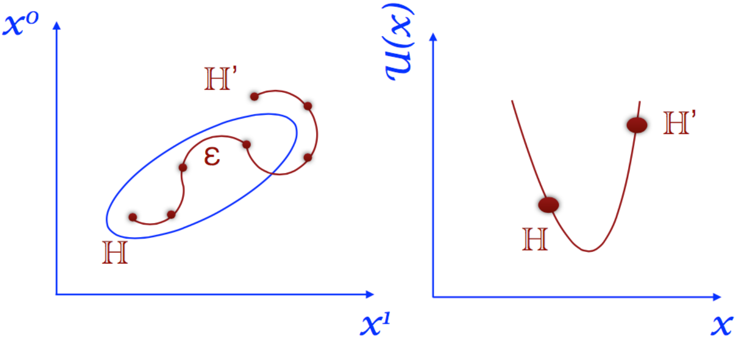
HMC: 5×10^3 points, acceptance : 70%



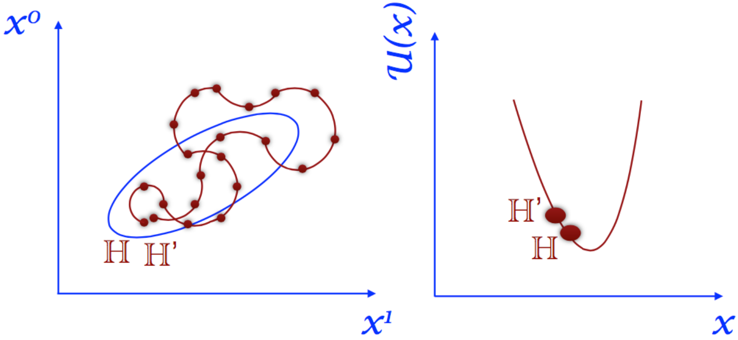
HMC challenges

- Tuning of the algorithm parameters
  - Step size
  - Number of steps per trajectory
  - Scaling
- Gradient of the log-likelihood computation is expensive (requires adequate fitting)
- Behavior with highly multi-modal posterior needs to be assessed

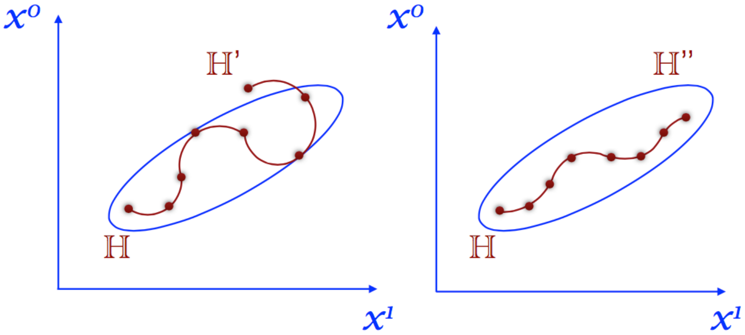
Effect of step size



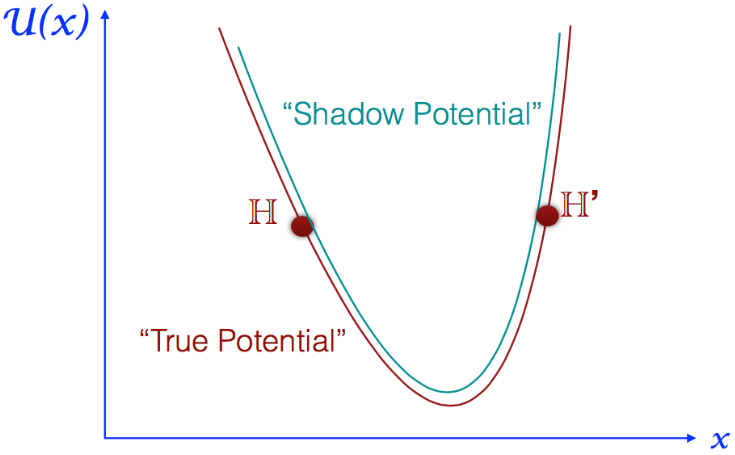
Effect of trajectory length



Effect of scaling and mass matrix



Fitting the gradient



Thank you!

Schemes

- Thermostated annealing

$$\gamma = \begin{cases} \frac{1}{2} & 0 \leq \rho \leq \rho_0 \\ \frac{1}{2}\left(\frac{\rho}{\rho_0}\right)^{-2} & \rho > \rho_0 \end{cases}, \qquad (2)$$

- Simulated annealing

$$\gamma = \begin{cases} \frac{1}{2}10^{-\xi\left(1-\frac{i}{t_{cool}}\right)} & 0 \leq i \leq t_{cool} \\ \frac{1}{2} & i > t_{cool} \end{cases}, \qquad (3)$$

- Inertia annealing

$$w(i) = \begin{cases} w_f 10^{\log_{10}(\frac{w_i}{w_f})(1-\frac{i}{T_w})} & \text{if} \quad 0 \leq i \leq T_w \\ w_f & \text{if} \quad i > T_w, \end{cases} \qquad (4)$$

Likelihood and Bayesian data analysis test

- Measured signal s that contains a gravitational wave h and noise n:

$$s(t) = h(t) + n(t)$$

- Use Bayesian statistics

$$P(h \mid s) = \frac{P(s \mid h)P(h)}{P(s)}$$

- Definition of a noise weighted scalar product for the expression of the likelihood

$$\langle h_1 | h_2 \rangle = 2 \int_0^\infty \frac{\widetilde{h_1}(\lambda_1^\mu)\widetilde{h_2}^*(\lambda_2^\mu) + \widetilde{h_1}^*(\lambda_1^\mu)\widetilde{h_2}(\lambda_2^\mu)}{S_n(f)} \mathrm{d}f$$

Likelihood - SNR

$$\ln \mathcal{L}_R = \langle s|h \rangle - \frac{1}{2}\langle h \mid h \rangle$$

$$\mathrm{SNR} = \frac{\langle s|h \rangle}{\sqrt{\langle h \mid h \rangle}}$$

Fisher matrix

$$F_{\mu\nu} = \left\langle \frac{\partial h}{\partial \lambda^\mu} \middle| \frac{\partial h}{\partial \lambda^\nu} \right\rangle$$

Particle Swarm Optimization (PSO)

- **Idea**: Define a swarm of particles evolving according to movement equations in the parameter space
- PSO equations:

$$
\begin{aligned}
X^i(t_{j+1}) &= X^i(t_j) + V^i(t_j), \\
V^i(t_{j+1}) &= w V^i(t_j) + c_1 \xi_1 (P^i(t_j) - X^i(t_j)) \\
&+ c_2 \xi_2 (G(t_j) - X^i(t_j))
\end{aligned}
$$

- Parameters:
  - inertia w
  - personal c_1
  - *social* c_2
  - $\xi_1, \xi_2 \in \mathcal{U}(0,1)$
  - Number of particles N_{part}

Differential Evolution (DE)

- **Idea**: Construct a mutant solution using previous position and a differential vector constructed with two other particles of the swarm
- DE proposed moves:

$$X^i(t_{j+1}) = X^i(t_j) + \gamma \left[X^j(t_j) - X^k(t_j) \right] \quad (5)$$

  - $i \neq j \neq k \in [1, .., N_p]$
  - differential weight $\gamma = 2.38/\sqrt{2D}$ with D being the dimension of the search space parameter

- Jumps accepted with probability α where $\alpha = min(1, H)$ and H_M is the Metropolis ratio

$$H_M = \frac{\mathcal{L}(X^i(g+1))}{\mathcal{L}(X^i(g))}, \quad (6)$$

MCMC based algorithms

- **Idea**: Stochastic moves of particles using jump proposal adopted to the problem. Jumps are accepted according to a given criterion
- Jumps proposal: normal distributed jumps in eigendirections of $\Gamma_{\mu\nu}$ with standard deviations $\sigma_\mu = 1/\sqrt{DE_\mu}$
- Acceptance criterion
  - Metropolis-Hastings ratio: moves are accepted with probability α where $\alpha = min(1, H)$ and H is

$$H = \frac{\pi(x')p(s|x')q(x|x')}{\pi(x)p(s|x)q(x'|x)}. \tag{7}$$

  - Greedy criterion (Uphill Climber UC): $\mathcal{L}(\lambda^{new}) > \mathcal{L}(\lambda^{old})$
- MCMC algorithms applied to the $P^i(t_j)$

First data set - Residuals

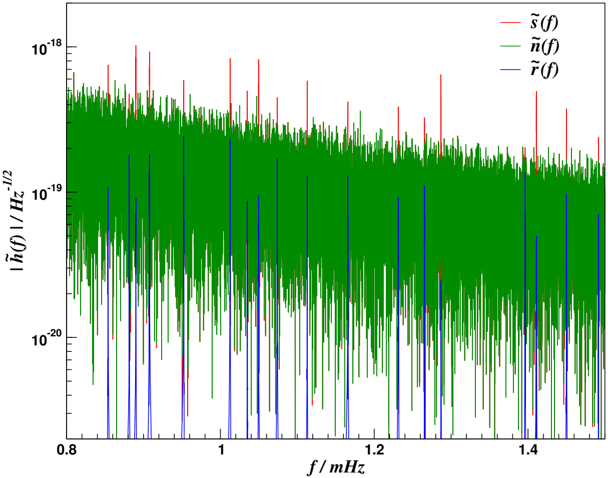


Figure: A plot of the power spectra for the injected data set, the instrumental noise and the residual for data set 1.

Second data set - Residuals

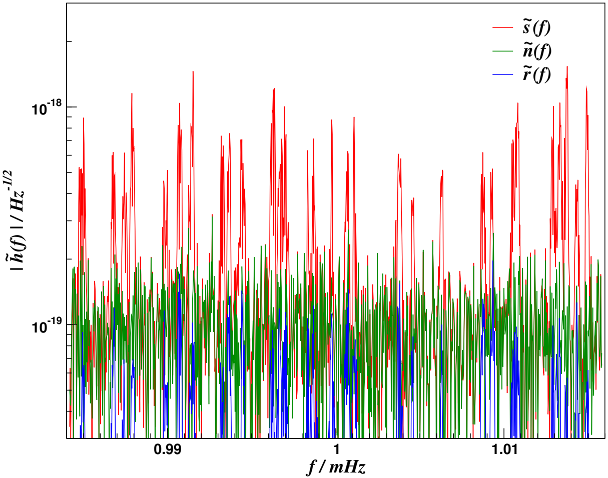


Figure: A plot of the power spectra for the injected data set, the instrumental noise and the residual for data set 2.