Journée des doctorants – APC – 10 Novembre 2016

The Gauge-Gravity duality
&
The Renormalization Group

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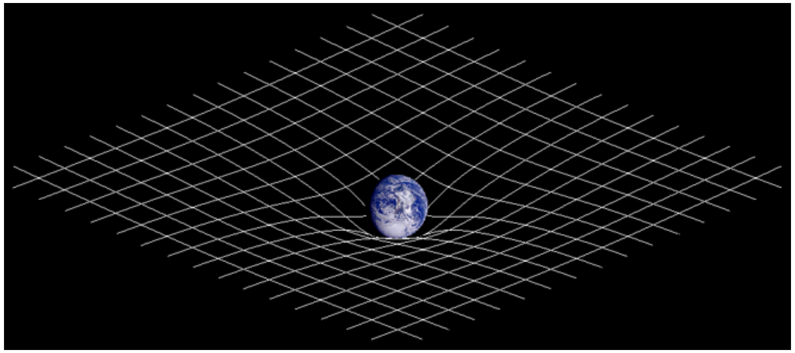
The Gauge-Gravity duality

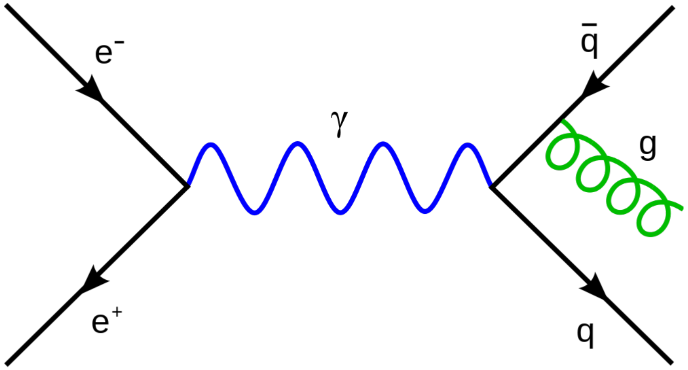
The Gauge-Gravity duality

Gravitational theories
in
d+1 dimensions

$\longleftrightarrow$

Gauge theories
in
d dimensions

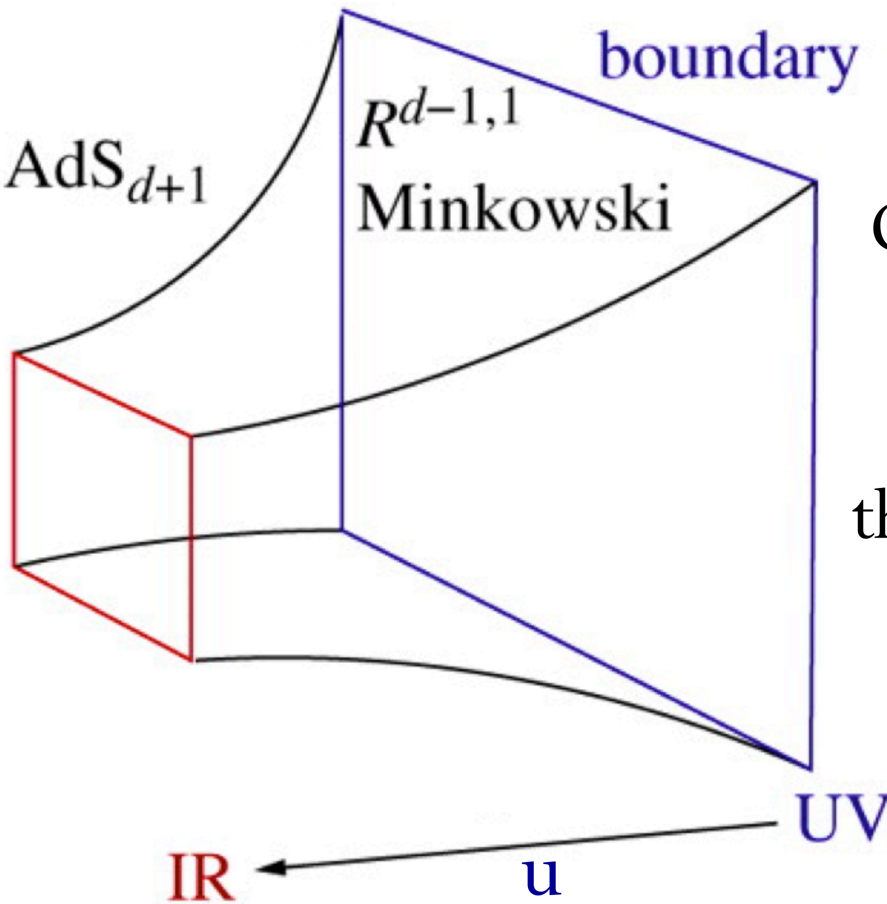




AdS/CFT correspondence

The AdS/CFT correspondence

Field theory in AdS_{d+1}



Conformal Field Theory (CFT) living on the AdS boundary

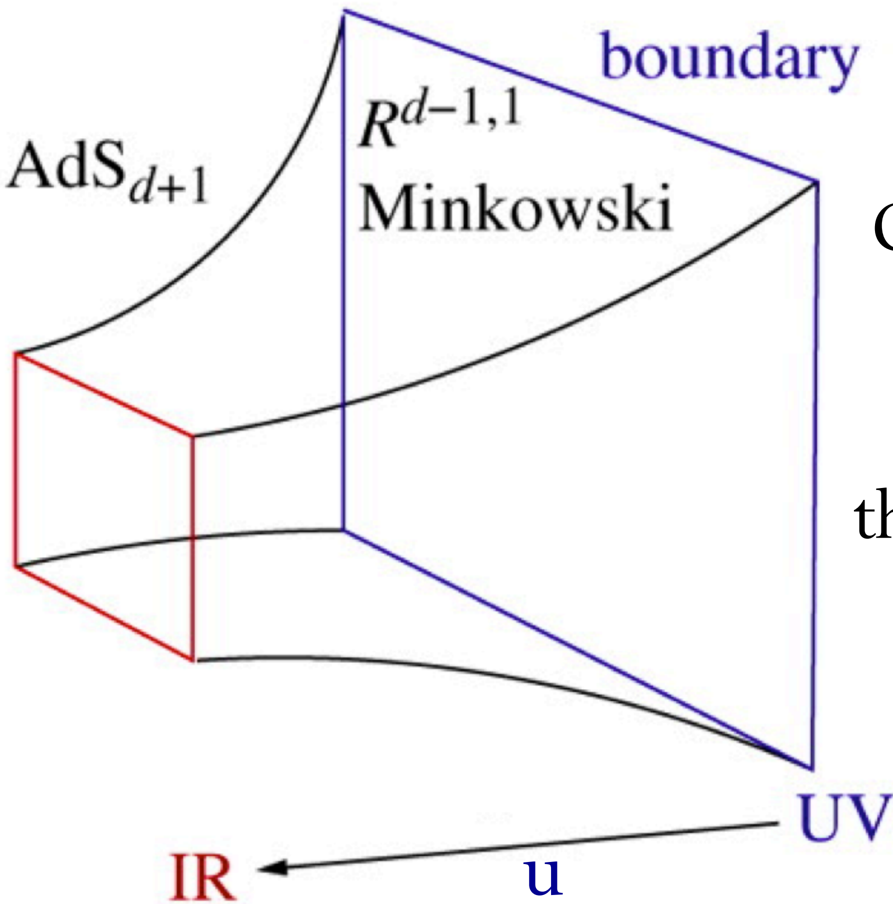
Adapted from Faulkner et al, Phil. Trans. R. Soc. A 2011 369 1640-1669.

The AdS/CFT correspondence

The scale of $R^{1,d-1}$ changes along the holographic direction.

u is the holographic coordinate

Field theory in AdS_{d+1}



Conformal Field Theory (CFT) living on the AdS boundary

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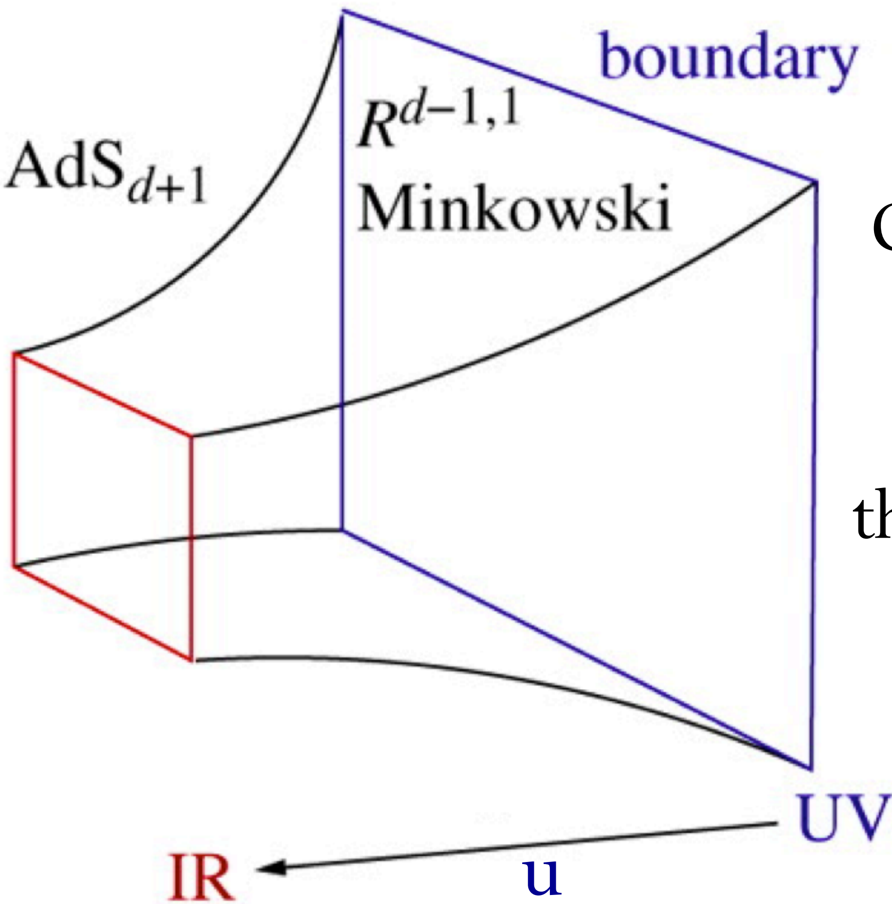
The AdS/CFT correspondence

Moving towards the interior :
decreasing the energy scale

Field theory in AdS_{d+1}



Conformal Field Theory (CFT) living on the AdS boundary

Adapted from Faulkner et al, Phil. Trans. R. Soc. A 2011 369 1640-1669.

AdS/CFT correspondence

There is a one-to-one correspondence between:

$\mathbf{AdS_{d+1}}$ Bulk field	$\longleftrightarrow$	**$\mathbf{CFT_d}$** Gauge invariant, single-trace operators at d-dim Minkowski space

The AdS/CFT correspondence

In particular:

Scalar operator $\mathcal{O}(x)$ $\longleftrightarrow$ Scalar field $\phi(x,u)$

Vector operator $J_\mu(x)$ $\longleftrightarrow$ Vector field $A_\mu(x,u)$

Stress-en. tensor $T_{\mu\nu}(x)$ $\longleftrightarrow$ Metric $g_{\mu\nu}(x,u)$

The AdS/CFT correspondence

Operator $\mathcal{O}(x)$ with conformal dimension Δ.

ϕ_0 : source for the operator $\mathcal{O}(x)$

$$\boxed{\left\langle 0 \middle| e^{-\int d^d x\, \phi_0(x)\mathcal{O}(x)} \middle| 0 \right\rangle_{CFT_d} = e^{-I_{GRA}(\phi_0)}}$$

$I_{\mathrm{GRA}}(\phi_0)$: on-shell action in (d+1) dimensional gravity

ϕ_0 is a boundary condition on $\phi(x,u)$:

$$\phi_0(x) = e^{\mathrm{u}(\Delta-\mathrm{d})/\mathrm{l}}\phi(x,u)\Big|_{u\to-\infty}$$

The AdS/CFT correspondence

The precise map between physical observables in the two sides of the correspondence is given by the following formula:

$$\boxed{\left\langle 0 \left| e^{-\int d^d x\, \phi_0(x) \mathcal{O}(x)} \right| 0 \right\rangle_{CFT_d} = e^{-I_{SUGRA}(\phi_0)}}$$

ϕ_0 : source for the operator $\mathcal{O}(x)$

$I_{\text{sugra}}(\phi_0)$: on-shell action in supergravity

The Gauge-Gravity duality

Gravitational theories
in
d+1 dimensions

$\longleftrightarrow$

Gauge theories
in
d dimensions

Interpretation of the extra dimension:

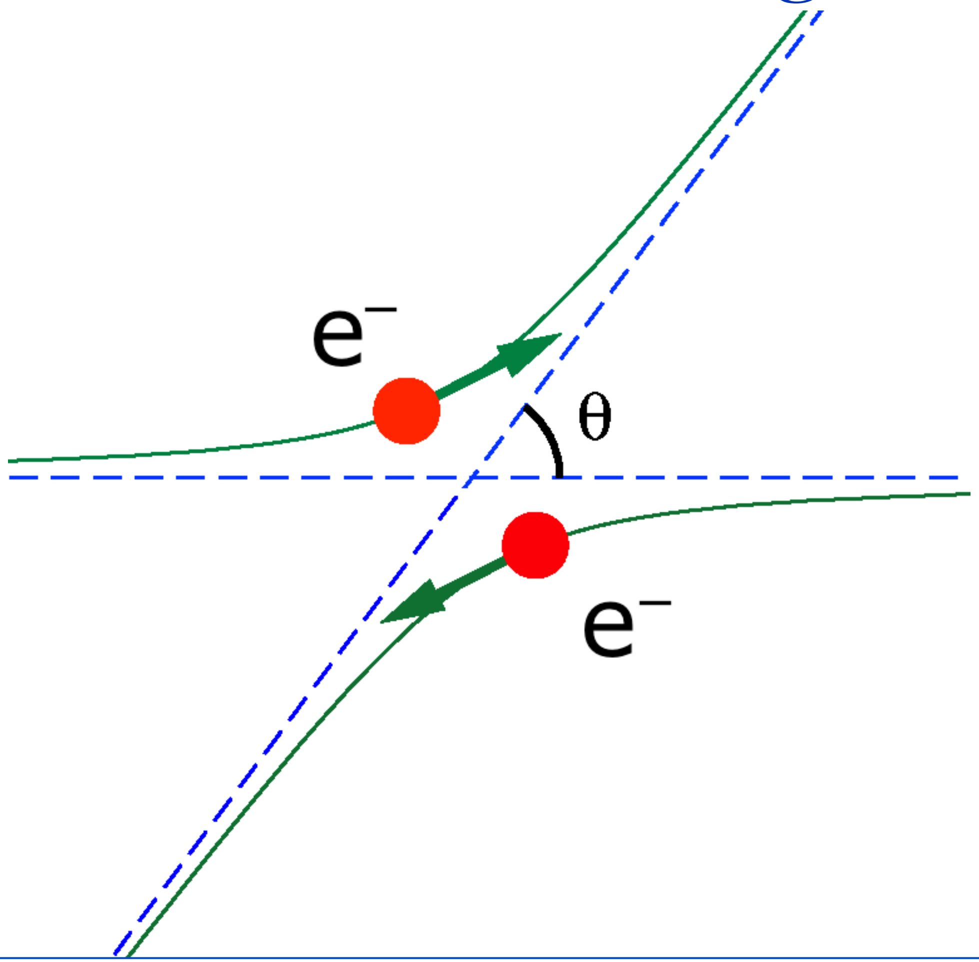
Fields changing in the extra dimension

$\longleftrightarrow$

Running couplings in the Quantum Field Theory

Running couplings

Running couplings



$$\alpha = \frac{e^2}{2\varepsilon_0 hc}$$

Well verified QFT prediction:

$$\alpha = \alpha(\text{energy})$$

The couplings flow with a change in the energy scale: Renormalization Group (RG) Flow

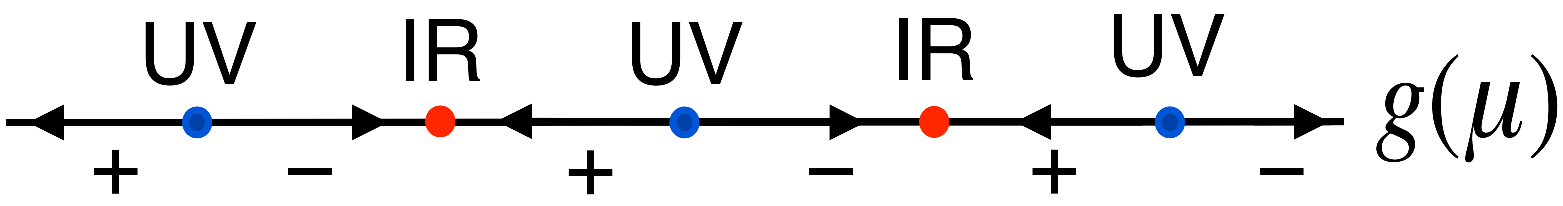
Renormalization Group Flow

The RG flow is given by a set of 1st order equations.

In the case of a single coupling g:

$$\frac{dg}{d\log\mu} = \beta(g)$$

The direction of the flow is given by the sign of β.



1st order: flows are localized between consecutive zeroes of β, fixed points cannot be skipped.

Gravitational dual of RG flows

Gravitational theories in d+1 dimensions	$\longleftrightarrow$	Gauge theories in d dimensions

There is a correspondence between:

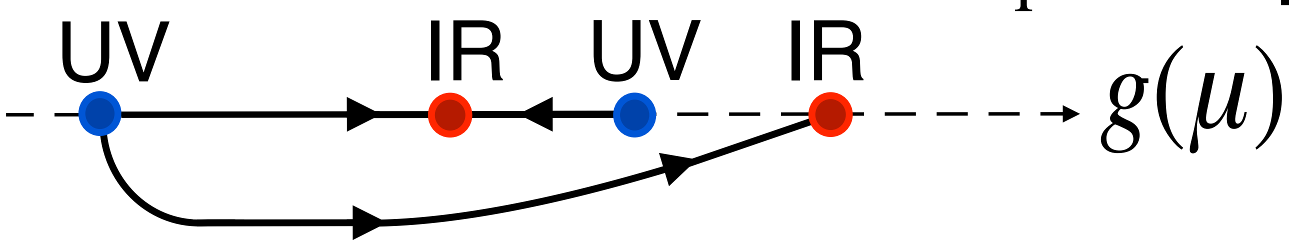
Holographic RG flows	$\longleftrightarrow$	Quantum Field Theory RG flows

Holographic RG flows

Holographic RG flows are given by
2nd order differential equations.

For a single coupling we characterized and classified all solutions that correspond to asymptotically AdS space-times.

2nd order : the flow does not need to stop when $\beta=0$.



Are these flows physical?

The Setup

$$S[g,\varphi] = \int d^{d+1}x\sqrt{-g}\left(R - \frac{1}{2}\partial_a\phi\partial^a\phi - V(\phi)\right)$$

Generic V(ϕ)

Consider the most general form of a solution that preserves Poincaré invariance:

$$ds^2 = du^2 + e^{2A(u)}\eta_{\mu\nu}dx^\mu dx^\nu$$

$$\phi = \phi(u)$$

Energy scale:

$$\mu \equiv \mu_0 e^{A(u)}$$

Flow equations

The Einstein equations become:

$$2(d-1)\ddot{A}(u) + \dot{\phi}^2 = 0$$

$$d(d-1)\dot{A}(u)^2 - \frac{1}{2}\dot{\phi}^2 + V(\phi) = 0$$

There are 3 integration constants.

1st order formalism: Super-potential

$$\dot{A}(u) = -\frac{W(\phi)}{2(d-1)}, \quad \dot{\phi}(u) = \frac{d}{d\phi}W(\phi) \equiv W'$$

$$V(\phi) = \frac{1}{2}W'^2(\phi) - \frac{d}{4(d-1)}W^2(\phi)$$

With a sol. *W(ϕ)*, the equations for *A* and *ϕ* are 1st order.

There is one integration constant per equation.

Only one V, infinitely many W's parameterized by one integration constant: not all of the W(ϕ) are physical.

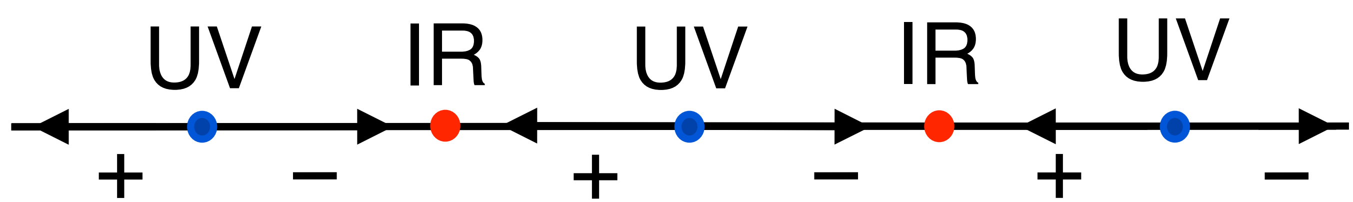
1st order formalism: Super-potential

Regularity seems to pick up a single $W(\phi)$ that corresponds to the effective potential on the QFT side.

$$\dot{A}(u) = -\frac{W(\phi)}{2(d-1)}, \qquad \dot{\phi}(u) = \frac{d}{d\phi}W(\phi) \equiv W'$$

Things now seem equiv. to QFT RG flows:

$$\beta(\phi) \equiv \frac{d\phi}{dA} = -2(d-1)\frac{W'(\phi)}{W(\phi)}$$



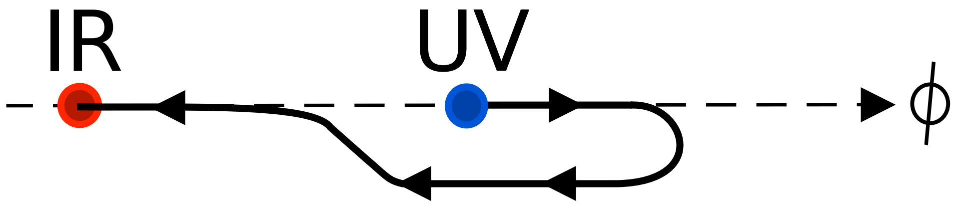
Non-perturbative treatment

We went beyond the small coupling limit and found non-perturbative, exotic holographic RG flows.

Some have no known Quantum Field Theory counterpart and suggest there is more on RG flows than we currently believe.

Flows reversing direction

Holographic RG flows may have turning points:

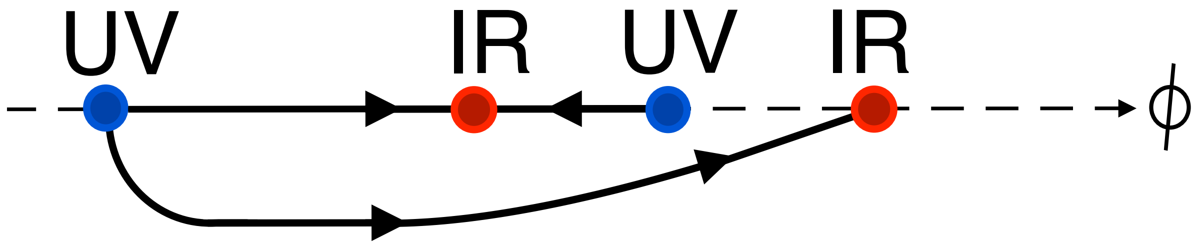


Multi-branched W(ϕ)

Multi-branched effective potential!

Skipping fixed points

Two RG flows from the same UV theory:



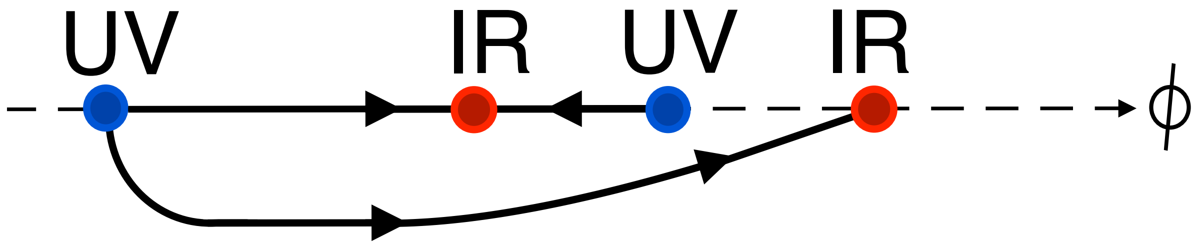
Same operator $\mathcal{O}(x)$

Different vacuum expectation values of $\mathcal{O}(x)$

Two non-perturbatively related RG flows

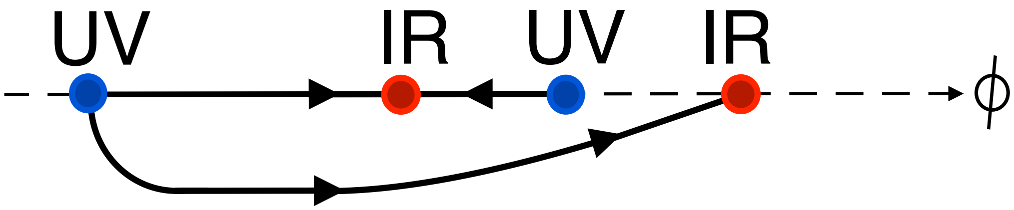
Skipping fixed points

Exotic flow: thermodynamically favorable



The solution skipping fixed points has lower free energy than the standard RG flow starting from the same UV fixed point.

Skipping fixed points



The solution skipping fixed points has lower free energy than the standard RG flow starting from the same UV fixed point.

Exotic flow: thermodynamically favorable

Skipping fixed points

There are arguments using QFT that the RG flow equations may become second order in some cases. These are called the "quantum RG flows".

arXiv:1305.3908 [hep-th]

It is not yet clear if this allows for skipping fixed points.

Conclusions

In holographic RG flows:

- Couplings can increase and then decrease along the flow.

- Fixed points can be skipped for a single coupling

There may be more in QFT RG flows than what we currently believe!

Invitation

If you want to understand it better
and know more:

PhDiderot

Organisation sociale des doctorants du bâtiment Condorcet

PhD seminar: **December 1st at 2 pm**

https://phdiderot.wordpress.com

Thank you for your attention !